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13. The temperature of a plate at the point (x, y) is $T(x, y) = 20 - 3x^2 - 2y^2$. Find the path that a heat seeking particle would travel if it starts at the point $(1, 4)$. (The particle always moves in the direction of the greatest increase in temperature.)

Let $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ be the path of the particle.

$\vec{r}\,'(t) = c(t)\,\vec{\nabla}T$ (because at each moment the direction of the particle is $\perp$ to which ever level set the particle is on.)

$x'(t)\vec{i} + y'(t)\vec{j} = c(t)\left(-6x\vec{i} - 4y\vec{j}\right)$

$\frac{x'(t)}{-6x(t)} = c(t) \quad \frac{y'(t)}{-4y(t)} = c(t)$

So $\int \frac{2}{3}\frac{x'(t)}{x(t)}\,dt = \int \frac{y'(t)}{y(t)}\,dt$

$\frac{2}{3}\ln|x(t)| + k = \ln|y(t)|$

exponentiate

$\pm e^k x^{\frac{2}{3}} = y$

Let $\pm e^k = K$

$Kx^{\frac{2}{3}} = y$

Plug in $(1, 4)$

$K(1) = 4$

$y = 4x^{\frac{2}{3}}$

14. Find $\int\int_R e^{x^2+y^2}\,dA$, where R is the region inside $x^2 + y^2 = 9$.

$\int_0^{2\pi}\int_0^3 re^{r^2}\,dr\,d\theta = \frac{1}{2}e^{r^2}\Big]_0^3 \cdot 2\pi = \frac{1}{2}\left(e^9 - 1\right)2\pi$