10 2

11. Find the directional derivative of $f(x,y) = y^3 \ln x$ at the point $(1,2)$ in the direction of $\vec{u} = \frac{1}{\sqrt{2}}(\vec{i} - \vec{j})$. Notice $|\vec{u}| = 1$

$$D_{\vec{u}} f|_{pt} = \vec{\nabla} f|_{pt} \cdot \vec{u} = \left(\frac{y^3}{x}\vec{i} + 3y^2 \ln x\, \vec{j}\right)\Big|_{(1,2)} \cdot \vec{u} = (8\vec{i} + 0\vec{j}) \cdot \left(\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right)$$

$$= \frac{8}{\sqrt{2}}$$

12. Find all local maximum points, all local minimum points, and all saddle points of $f(x,y) = x^3 + y^3 - 6xy$.

$f_x = 3x^2 - 6y$

$f_y = 3y^2 - 6x$

Both partials vanish when

$y = \frac{1}{2}x^2$

and $3y^2 - 6x = 0$

So $y = \frac{1}{2}x^2$

and $\frac{3}{4}x^4 - 6x = 0$

$3x^4 - 24x = 0$

$3x(x^3 - 8) = 0$

So $x = 0$ or $x = 2$

and $y = \frac{1}{2}x^2$

The critical points are

$(0,0)$ $(2,2)$

$f_{xx} = 6x$ $f_{xy} = -6$

$f_{yy} = 6y$

at $(0,0)$:

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= 0 - 36 < 0$

so $(0,0,0)$ is a saddle point

at $(2,2)$

$D = 12 \cdot 12 - (36) > 0$

$f_{xx} = 12 > 0$

$(2, 2, f(2,2))$ is a local min