18

9. Where do the following two lines intersect? CHECK YOUR ANSWER!

$$\frac{x-1}{1}=\frac{y-8}{2}=\frac{z-8}{3} \quad \text{and} \quad \frac{x+4}{-1}=\frac{y-10}{2}=\frac{z+4}{-2}$$

Walk on the left line $x=1+t$ $y=8+2t$ $z=8+3t$. What time puts you on the second line?

$\frac{5+t}{-1}=\frac{2t-2}{2}=\frac{12+3t}{-2}$

I need $\begin{cases} 10+2t=-2t+2 \\ -10-2t=-12-3t \end{cases}$

I need $\begin{cases} 4t=-8 \\ +2=-t \end{cases}$

So $t=-2$

The intersection is

$x=-1$
$y=4$
$z=2$

$(-1,4,2)$ is on the first line because

$\frac{-2}{1}=\frac{4-8}{2}=\frac{-6}{3}$ ✓

$(-1,4,2)$ is on the second line because

$\frac{3}{-1}=\frac{-6}{2}=\frac{6}{-2}$ ✓

10. Find the length of the curve $\overrightarrow{r}(t)=t^2\overrightarrow{i}+t^3\overrightarrow{j}$ for $1\le t\le 2$.

$$\int_{1}^{2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_{1}^{2}\sqrt{(2t)^2+(3t^2)^2}\,dt=\int_{1}^{2}\sqrt{4t^2+9t^4}\,dt$$

$$=\int_1^2 t\sqrt{4+9t^2}\,dt = \frac{1}{18}\frac{2}{3}(4+9t^2)^{\frac{3}{2}}\Big]_1^2$$

$$=\frac{1}{27}\left((40)^{\frac{3}{2}}-(13)^{\frac{3}{2}}\right)$$