7. (There is no partial credit for this problem. Make sure your answer is correct.) Let $\vec{a} = 2\vec{i} + 4\vec{j} + 6\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} + \vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$.

$\vec u = \text{proj}_{\vec a}\vec b = \frac{\vec a \cdot \vec b}{\vec a \cdot \vec a}\vec a = \frac{6+16+6}{4+16+36}\vec a = \frac{28}{56}\vec a = \frac{1}{2}(2\vec i + 4\vec j + 6\vec k) = \vec i + 2\vec j + 3\vec k = \vec u$

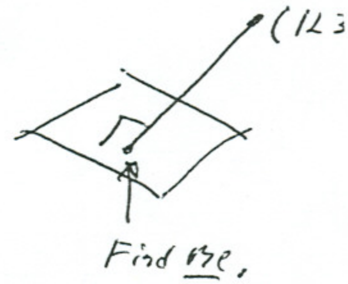
$\vec v = \vec b - \vec u = (3\vec i + 4\vec j + \vec k) - (\vec i + 2\vec j + 3\vec k) = 2\vec i + 2\vec j - 2\vec k = \vec v$

check $\vec u \parallel \vec a$

$\vec u + \vec v = \vec a$ ✓

$\vec v \cdot \vec a = 4 + 8 - 12 = 0$ ✓

8. Find the point on $2x + 3y + 4z = 49$ which is closest to $(1, 2, 3)$.



The line $\perp$ $2x+3y+4z=49$ through $(1,2,3)$ is

$x = 1+2t$
$y = 2+3t$
$z = 3+4t$

This line hits the plane at

$2(1+2t)+3(2+3t)+4(3+4t)=49$
$2+4t+6+9t+12+16t=49$
$29t = 29$
$t=1$

Pt is $(3,5,7)$