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5. What are the equations of the line tangent to the curve which is parameterized by $\vec{r}(t) = (3t^3 + 2t)\vec{i} + 6t^2\vec{j} + 4t^3\vec{k}$ at $t = 1$?

$$\begin{aligned}\vec{r}(1) &= 5\vec{i} + 6\vec{j} + 4\vec{k} \\ \vec{r}'(t) &= (9t^2 + 2)\vec{i} + 12t\vec{j} + 12t^2\vec{k} \\ \vec{r}'(1) &= 11\vec{i} + 12\vec{j} + 12\vec{k}\end{aligned}$$

Tan line:

$$\begin{cases} x = 5 + 11u \\ y = 6 + 12u \\ z = 4 + 12u \end{cases}$$

6. Find the equation of the plane tangent to the surface $z = x^2 + 3y^3$ at the point where $x = 3$ and $y = -1$. The z -coordinate is $9 - 3 = 6$

View the surface as the level set $0 = x^2 + 3y^3 - z$

$\vec{\nabla}$'s are $\perp$ to level sets

$$\vec{\nabla}(\text{RHS}) = 2x\vec{i} + 9y^2\vec{j} - \vec{k}$$

$$\vec{\nabla}(\text{RHS})|_{P_1} = 6\vec{i} + 9\vec{j} - \vec{k}$$

$$6(x-3) + 9(y+1) - (z-6) = 0$$