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19. Find a function $f(x, y)$ with $\vec{\nabla} f = (y^2 + 2xy)\vec{i} + (x^2 + 2xy + 3y^2)\vec{j}$.

$$f_x = y^2 + 2xy$$

$$f = xy^2 + x^2y + c(y)$$

$$f_y = 2xy + x^2 + c'(y)$$

$$f_y \text{ also} = x^2 + 2xy + 3y^2$$

$$\text{so } c'(y) = 3y^2$$

$$c(y) = y^3$$

$$\text{so } f = xy^2 + x^2y + y^3$$