164 ~~144~~ ~~103~~

15. The temperature of a plate at the point (x, y) is $T(x, y) = 20 - x^2 - 2y^2$. Find the path that a heat seeking particle would travel if it starts at the point $(-1, 2)$. (The particle always moves in the direction of the greatest increase in temperature.)

The particle travels on $2x^2 = y$

$r(t) = x(t)\vec{i} + y(t)\vec{j}$ is the position of the object at time t

$x(0) = -1 \quad y(0) = 2$

$\vec{\nabla} T = -2x\vec{i} - 4y\vec{j}$

$\vec{r}\,'(t) = c(t)\,\vec{\nabla} T|_{(x(t), y(t))}$

$x'(t)\vec{i} + y'(t)\vec{j} = c(t)\left(-2x(t)\vec{i} - 4y(t)\vec{j}\right)$

$\frac{x'(t)}{-2x(t)} = c(t) = \frac{y'(t)}{-4y(t)}$

$2\int \frac{x'(t)}{x(t)}\,dt = \int \frac{y'(t)}{y(t)}\,dt$

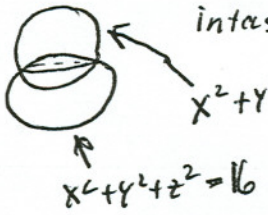
$2\ln|x(t)| = \ln|y(t)| + C$

$x(t)^2 = K y(t)$

at $t = 0 \quad +1 = K \quad 2$

$K = \frac{1}{2}$

16. Find the volume of the solid which is inside both $x^2 + y^2 + z^2 = 16$ and $x^2 + y^2 + (z-6)^2 = 16$ is a solid in 3− space.



intersection is $r^2 + z^2 = 16$

$x^2 + y^2 + (z-6)^2 = 16 \quad r^2 + (z-6)^2 = 16$

$x^2 + y^2 + z^2 = 16$

So $z^2 = (z-6)^2$

$z = z - 6$ or $z = -(z-6)$

nonsense $\quad 2z = 6$

$z = 3$

$r = \sqrt{16-9} = \sqrt{7}$

$$\text{Vol} = \int_0^{2\pi}\int_0^{\sqrt{7}}\int_{6-\sqrt{16-r^2}}^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta = 2\pi\int_0^{\sqrt{7}} 2r\sqrt{16-r^2} - 6r\,dr = 2\pi\left(-\tfrac{2}{3}(16-r^2)^{\frac{3}{2}} - 3r^2\right)\Big|_0^{\sqrt{7}}$$

$= 2\pi\left(-\frac{2}{3}\cdot 9^3 - 3\cdot 7 + \frac{2}{3}\cdot 16^3\right)$

$= 2\pi\left(-39 + \frac{128}{3}\right)$

$= 2\pi\left(\frac{11}{3}\right)$