

13. Sand is ~~pouring~~ <sup>pouring</sup> onto a conical pile in such a way that at a certain instant the height is 100 inches and is increasing at 3 inches per minute and the radius is 40 inches and is increasing at 2 inches per minute. How fast is the volume increasing at that instant?

Let  $V$  = the Volume of the sand at time  $t$   
 $h$  = the ht of the sand at time  $t$   
 $r$  = the radius of the base at time  $t$

at our favorite time

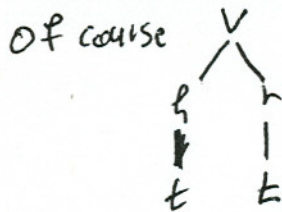
$$h = 100 \quad t = 40$$

$$\frac{dh}{dt} = 3 \quad \frac{dr}{dt} = 2$$

$$V = \frac{1}{3} \pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3} \pi r^2 \frac{dh}{dt} + \frac{2}{3} \pi r h \frac{dr}{dt}$$

$$\left. \frac{dV}{dt} \right|_{\text{our favorite time}} = \left( \frac{1}{3} \pi (40)^2 \cdot 3 + \frac{2}{3} \pi 40 \cdot 100(2) \right) \frac{\text{in}^3}{\text{sec}}$$



241 - chain rule works just as well

14. Find all local maximum points, all local minimum points, and all saddle points of  $f(x, y) = x^2 y - 6y^2 - 3x^2$ .

$$f_x = 2xy - 6x$$

$$f_y = x^2 - 12y$$

$$f_x = 0 \text{ and } f_y = 0 \text{ when}$$

$$y = \frac{x^2}{12}$$

$$0 = 2x \frac{x^2}{12} - 6x$$

$$0 = \frac{x}{6} (x^2 - 36)$$

$$x = 0, 6, -6$$

Critical pts are  $(0, 0), (6, 3), (-6, 3)$

$$f_{xx} = 2y - 6$$

$$f_{xy} = 2x$$

$$f_{yy} = -12$$

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

$$\text{at } (0, 0) \quad D = (0)(-12) - 0 > 0$$

$$f_{xx} = -6 < 0$$

$(0, 0, 0)$  is a local max

$$\text{at } (6, 3) \quad D = 0 - (12)^2 < 0$$

$$\text{at } (-6, 3) \quad D = 0 - (12)^2 < 0$$

$(0, 0, 0)$  is a local max

$(6, 3, f(6, 3))$  and  $(-6, 3, f(-6, 3))$  are saddle points