13. Sand is ~~poring~~ pouring onto a conical pile in such a way that at a certain instant the height is 100 inches and is increasing at 3 inches per minute and the radius is 40 inches and is increasing at 2 inches per minute. How fast is the volume increasing at that instant?

Let V = the Volume of the Sand at time t
h = the ht of the Sand at time t
r = the radius of the base at time t

at our favorite time
$h = 100$ $r = 40$
$\frac{dh}{dt} = 3$ $\frac{dr}{dt} = 2$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + \frac{2}{3}\pi r h \frac{dr}{dt}$$

$$\left.\frac{dV}{dt}\right|_{\text{our favorite time}} = \left[\frac{1}{3}\pi (40)^2 \cdot 3 + \frac{2}{3}\pi\, 40 \cdot 100 (2)\right] \frac{in^2}{sec}$$

Of course

241 - chain rule works just as well

14. Find all local maximum points, all local minimum points, and all saddle points of $f(x, y) = x^2y - 6y^2 - 3x^2$.

$f_x = 2xy - 6x$
$f_y = x^2 - 12y$
$f_x = 0$ and $f_y = 0$ when
$y = \frac{x^2}{12}$
$0 = 2x\frac{x^2}{12} - 6x$
$0 = \frac{x}{6}(x^2 - 36)$
$x = 0, 6, -6$
Critical pts are $(0,0)$, $(6,3)$, $(-6,3)$

$f_{xx} = 2y - 6$
$f_{xy} = 2x$
$f_{yy} = -12$

$D = f_{xx} f_{yy} - (f_{xy})^2$

at $(0,0)$ $D = (-6)(-12) - 0 > 0$
$f_{xx} = -6 < 0$
$(0, 0, 0)$ is a local max

at $(6,3)$ $D = 0 - (12)^2 < 0$

at $(-6,3)$ $D = 0 - (12)^2 < 0$

$(0,0,0)$ is a local max
$(6, 3, f(6,3))$ and $(-6, 3, f(-6,3))$ are saddle points