162

~~142~~ ~~103~~

11. Find the length of the curve $\vec{r}(t) = \frac{t^3}{3}\vec{i} + \frac{t^2}{2}\vec{j}$ for $0 \le t \le 1$.

$$\text{length} = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^1 \sqrt{(t^2)^2 + t^2}\, dt = \int_0^1 t\sqrt{t^2+1}\, dt$$

$$= \frac{1}{3}(t^2+1)^{\frac{3}{2}}\Big]_0^1 = \frac{1}{3}\left(2\sqrt{2} - 1\right)$$

12. Find the directional derivative of $f(x,y) = x^2 \ln y$ at the point $(1,2)$ in the direction of $\vec{a} = \vec{i} - \vec{j}$.

$$D_{\frac{\vec{a}}{\|\vec{a}\|}} f\Big|_{(1,2)} = (\vec{\nabla} f)\Big|_{(1,2)} \cdot \frac{\vec{i} - \vec{j}}{\sqrt{2}} = \left(2x \ln y\, \vec{i} + \frac{x^2}{y}\vec{j}\right)\Big|_{(1,2)} \cdot \frac{\vec{i} - \vec{j}}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\left(2\ln 2 - \frac{1}{2}\right)$$