21. Does there exist a function $f(x,y)$ such that $\vec{\nabla} f = (6xy+4)\,\vec{i} + (3x^2-4y)\,\vec{j}$? If the answer is yes, then find this function $f(x,y)$.

$M_y = 6x \quad N_x = 6x \quad M_y = N_x$ so f exists.

$f_x = 6xy + 4$

$f = 3x^2y + 4x + c(y)$

$f_y = 3x^2 + c'(y)$

$f_y = 3x^2 - 4y$

so $c'(y) = -4y$

$c(y) = -2y^2$

$f = 3x^2y + 4x - 2y^2$

22. Let C be the curve which starts at $(1,0)$; travels along the x-axis until $(2,0)$; travels around the upper part of the circle $x^2+y^2=4$ to $(-2,0)$; travels along the x-axis to $(-1,0)$; and finally travels along the upper part of the circle $x^2+y^2=1$ back to $(1,0)$. Compute $\int_C \underbrace{(2x^2+6y)}_{M}\,dx + \underbrace{(3x+4y^2)}_{N}\,dy$.

$\underset{\text{Green's thm}}{\overset{\uparrow}{=}} \iint_{\text{region}} N_x - M_y \, dx\,dy = \iint_{\text{region}} 3-6 \, dx\,dy = (-3)$ area of region

$= (-3)\left(\frac{1}{2}(\pi 4 - \pi)\right) = \frac{-9\pi}{2}$