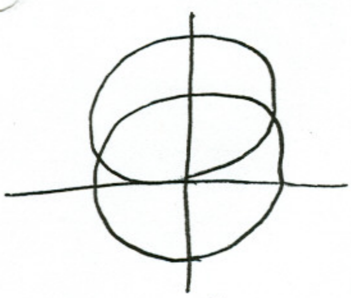
166 ~~107~~ 146

19. Find the area inside $r = 6\sin\theta$ and outside $r = 3$.



intersection $3 = 6\sin\theta$

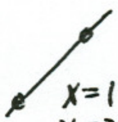
$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\int_{\pi/6}^{5\pi/6}\int_{3}^{6\sin\theta} r\,dr\,d\theta = \int_{\pi/6}^{5\pi/6} \frac{r^2}{2}\Big]_3^{6\sin\theta} d\theta = \frac{1}{2}\int_{\pi/6}^{5\pi/6}\left[\frac{36}{2}(1-\cos 2\theta) - 9\right] d\theta$$

$$= \frac{9}{2}\int_{\pi/6}^{5\pi/6} 1 - 2\cos 2\theta\, d\theta = \frac{9}{2}\left(\theta - \sin 2\theta\right)\Big]_{\pi/6}^{5\pi/6} = \frac{9}{2}\left(\frac{5\pi}{6} + \frac{\sqrt{3}}{2} - \frac{\pi}{6} + \frac{\sqrt{3}}{2}\right)$$

20. (16 points) Compute $\int_C 2y\,dx + 3x^2\,dy$, where C is the line segment from $(1,2)$ to $(3,4)$.



$x = 1 + 2t$

$y = 2 + 2t$

$$\text{Integral} = \int_0^1 2(2+2t)\,2\,dt + 3(1+2t)^2\,2\,dt$$

$$= \frac{4(2+2t)^2}{4} + (1+2t)^3\Big]_0^1$$

$$= (4)^2 + (3)^3 - (4+1)$$