16. (7 points) Suppose $\vec{r}''(t) = \vec{i} + e^t\vec{j}$, $\vec{r}'(0) = 2\vec{i} + \vec{j}$, and $\vec{r}(0) = \vec{i} + \vec{j}$. Find $\vec{r}(t)$.

$\vec{r}'(t) = t\vec{i} + e^t\vec{j} + \vec{C}_1$

$2\vec{i} + \vec{j} = \vec{r}'(0) = \vec{j} + \vec{C}_1$

$2\vec{i} = \vec{C}_1$

$\vec{r}'(t) = (t+2)\vec{i} + e^t\vec{j}$

$\vec{r}(t) = \left(\frac{t^2}{2} + 2t + C_2\right)\vec{i} + (e^t + C_3)\vec{j}$

$\vec{i} + \vec{j} = \vec{r}(0) = C_2\vec{i} + (1+C_3)\vec{j}$

so $C_2 = 1$ $C_3 = 0$

$\vec{r}(t) = \left(\frac{t^2}{2} + 2t + 1\right)\vec{i} + e^t\vec{j}$

17. (7 points) Where do the following two lines intersect? **CHECK YOUR ANSWER!**

$$\frac{x-4}{1} = \frac{y-2}{-1} = \frac{z-7}{2} \quad \text{and} \quad \frac{x+1}{-1} = \frac{y-10}{2} = \frac{z-6}{1}$$

walk on the line on the left

$\begin{cases} x = 4+t \\ y = 2-t \\ z = 7+2t \end{cases}$

You are on the other line when

$\frac{4+t+1}{-1} = \frac{2-t-10}{2} = \frac{7+2t-6}{1}$

i.e. $\begin{cases} 2(t+5) = -(-t-8) \end{cases}$

and $\; t+5 = -(2t+1)$

i.e. $\begin{cases} 2t+10 = t+8 \\ t+5 = -2t-1 \end{cases}$

i.e. $\begin{cases} t = -2 \\ 3t = -6 \end{cases}$

So $t = -2$

The point is

$(2,4,3)$

$(2,4,3)$ is on the line on the left

$\frac{2-4}{1} = \frac{4-2}{-1} = \frac{3-7}{2}$

$(2,4,3)$ is on the line on the right

$\frac{2+1}{-1} = \frac{4-10}{2} = \frac{3-6}{1}$