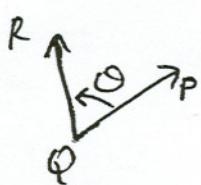*194*

14. (7 points) Consider the triangle with vertices $P = (1, 2, 3)$, $Q = (0, 2, 1)$, and $R = (4, 2, 7)$. Find the angle of this triangle at the vertex Q.

$$\overrightarrow{QP} = \vec{i} + 2\vec{k}$$

$$\overrightarrow{QR} = 4\vec{i} + 6\vec{k}$$

$$\overrightarrow{QP} \cdot \overrightarrow{QR} = \|\overrightarrow{QP}\| \, \|\overrightarrow{QR}\| \cos\theta$$

$$4 + 12 = \sqrt{5} \sqrt{52} \cos\theta$$

$$\boxed{\cos^{-1}\left(\frac{16}{\sqrt{5} \, \sqrt{52}}\right) = \theta}$$

15. (7 points) Find the directional derivative of $f(x, y) = x^3 \ln y$ at the point $(1, 2)$ in the direction of $\vec{u} = \frac{1}{\sqrt{2}}(\vec{i} + \vec{j})$.

$$D_{\vec{u}} f\Big|_{(1,2)} = \vec{\nabla} f\Big|_{(1,2)} \cdot \vec{u} = \frac{1}{\sqrt{2}}\left(3x^2 \ln y \, \vec{i} + \frac{x^3}{y}\vec{j}\right)\Big|_{(1,1)} \cdot (\vec{i} + \vec{j})$$

$$= \boxed{\frac{1}{\sqrt{2}}\left(3\ln 2 + \frac{1}{2}\right)}$$