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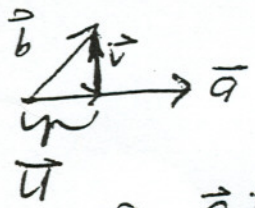
10. (8 points) Let $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = 5\vec{i} + 2\vec{j} + 2\vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$. **CHECK YOUR ANSWER!**



$\vec{u} = \text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{5+2+2}{1+1+1}\vec{a} = 3\vec{a}$

$\vec{u} = 3\vec{i} + 3\vec{j} + 3\vec{k}$

$\vec{v} = \vec{b} - \vec{u} = 5\vec{i} + 2\vec{j} + 2\vec{k} - (3\vec{i} + 3\vec{j} + 3\vec{k})$

$\vec{v} = 2\vec{i} - \vec{j} - \vec{k}$

Check $\vec{u} \parallel \vec{a}$ ✓

$\vec{u} + \vec{v} = \vec{b}$ ✓

$\vec{a}\cdot\vec{v} = 6 - 3 - 3 = 0$ ✓

11. (8 points) Find $\int\int_R e^{x^2+y^2}\,dA$, where R is the region inside $x^2 + y^2 = 16$.

$$\text{integral} = \int_0^{2\pi}\int_0^4 r e^{r^2}\,dr\,d\theta = \left(\frac{1}{2}e^{r^2}\Big]_0^4\right)\theta\Big]_0^{2\pi}$$

$$= (e^{16} - 1)\pi$$