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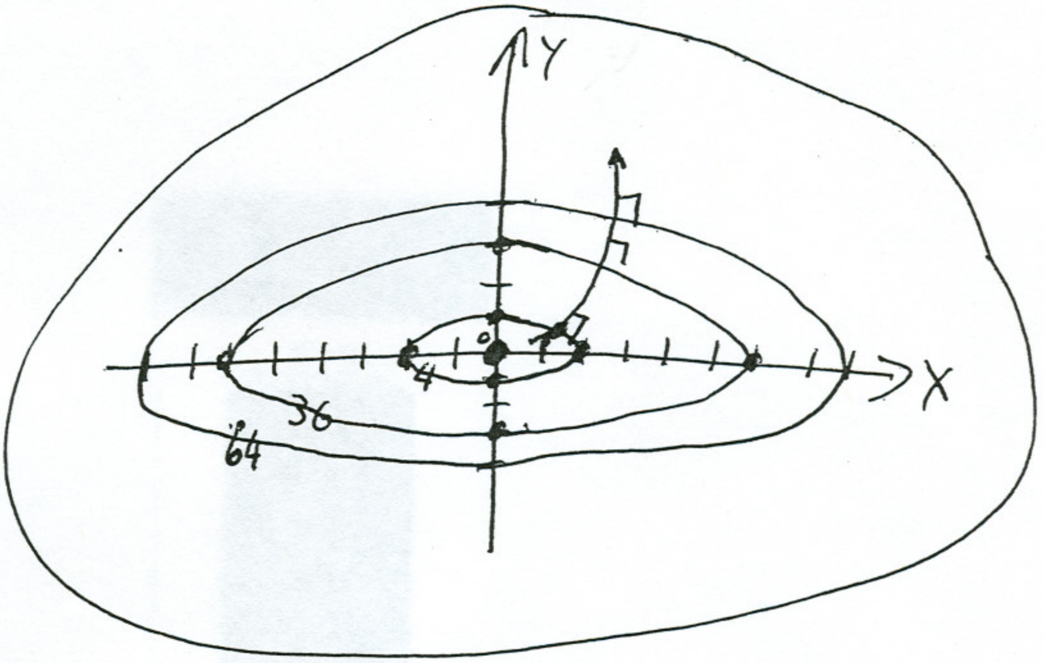
9. (14 points) The temperature of a plate at the point (x, y) is $T(x,y) = x^2 + 4y^2$.
(a) Draw and label the level sets $T = 0$, $T = 4$, $T = 36$, $T = 64$.

$0 = x^2+4y^2$ is a point

$4 = x^2+4y^2$ is $1 = \frac{x^2}{4} + y^2$

$36 = x^2+4y^2$ is $1 = \frac{x^2}{36} + \frac{y^2}{9}$

$64 = x^2+4y^2$ is $1 = \frac{x^2}{64} + \frac{y^2}{16}$



(b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point $(\sqrt{3}, \frac{1}{2})$. Draw the path of the particle.
(c) Find the equation which gives the path of the particle of part (b).

Let $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ be the path of the particle

We know that $\vec{r}'(t) = c(t)\, \vec{\nabla} T|_t$

$x'(t)\vec{i} + y'(t)\vec{j} = c(t)\left(2x(t)\vec{i} + 8y(t)\vec{j}\right)$

$\frac{x'(t)}{2x(t)} = c(t) = \frac{y'(t)}{8y(t)}$

$\therefore\ 4\frac{x'(t)}{x(t)} = \frac{y'(t)}{y(t)}$

Apply $\int - dt$ to both sides

$4\ln|x(t)| + R = \ln|y(t)|$

Apply e^{-} to both sides

$e^R\left(e^{\ln|x(t)|}\right)^4 = e^{\ln|y(t)|}$

$e^R (x(t))^4 = |y(t)|$

so $K x^4 = y$ when $K = \pm e^R$

But $(\sqrt{3}, \frac{1}{2})$ is on the curve so

$K 9 = \frac{1}{2}$

$K = \frac{1}{18}$

$y = \frac{1}{18}x^4$