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13. Find all local maximum points, all local minimum points, and all saddle points of $f(x, y) = x^2y - 6y^2 - 3x^2$.

$$\begin{aligned} f_x &= 2xy - 6x \\ f_y &= x^2 - 12y \\ f_x = 0 \text{ and } f_y = 0 \text{ when} \\ 2x(y-3) &= 0 \\ \text{and } x^2 - 12y &= 0 \\ \text{so } x=0 \text{ or } y=3 \text{ and } x=0 \text{ or } y=3 \\ f_{xx} &= 2y - 6 \\ f_{xy} &= 2x \\ f_{yy} &= -12 \\ D &= f_{xx}f_{yy} - (f_{xy})^2 \\ D &= (2y-6)(-12) - (2x)^2 \\ \text{at } (0,0) \quad D &= 6 \cdot 12 > 0 \\ f_{yy} &= -12 < 0 \end{aligned}$$

local max at $(0,0)$
at $(6,3)$ and $(-6,3)$
 $D = -144 < 0$
 $(6,3, f(6,3))$ and $(-6,3, f(-6,3))$
are saddle points.

14. The temperature of a plate at the point (x, y) is $T(x, y) = 20 - x^2 - 2y^2$. Find the path that a heat seeking particle would travel if it starts at the point $(-1, 2)$. (The particle always moves in the direction of the greatest increase in temperature.)

$\vec{r} = x(t)\vec{i} + y(t)\vec{j}$ is the path the particle

$$\begin{aligned} \vec{r}'(t) &= c(t) \vec{\nabla} T \\ x' \vec{i} + y' \vec{j} &= c(t) \cdot (-2x\vec{i} + 4y\vec{j}) \end{aligned}$$

$$\begin{aligned} \text{so } x' &= -2cx \\ y' &= 4cy \end{aligned}$$

$$\therefore \frac{x'}{-2x} = \frac{y'}{4y}$$

$$\therefore 2 \frac{x'}{x} = \frac{y'}{y}$$

$$\therefore 2 \ln|x| = \ln|y|$$

$$e^k (e^{\ln|x|})^2 = |y|$$

$$y = \pm e^k x^2$$

$$\text{Plugging } (-1, 2)$$

$$2 = (e^k) 1$$

$$y = 2x^2$$