~~153~~
173

11. Find the directional derivative of $f(x, y) = x^3 \ln y$ at the point $(1, 2)$ in the direction of $\vec{a} = \frac{1}{\sqrt{2}}(\vec{i} - \vec{j})$.

$$D_{\vec{a}} f \Big|_{(1,2)} = \vec{\nabla} f\Big|_{(1,2)} \cdot \vec{a} = \left(3x^2 \ln y \,\vec{i} + \frac{x^3}{y}\vec{j}\right)\Big|_{(1,2)} \cdot \vec{a}$$

$$= \frac{1}{\sqrt{2}} \left(3\ln 2\,\vec{i} + \tfrac{1}{2}\vec{j}\right) \cdot \left(\vec{i} - \vec{j}\right)$$

$$= \frac{1}{\sqrt{2}}\left(3\ln 2 - \frac{1}{2}\right)$$

12. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 200 inches and is increasing at 4 inches per minute and the radius is 50 inches and is increasing at 3 inches per minute. How fast is the volume increasing at that instant?

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + \frac{2}{3}\pi r \frac{dr}{dt} h$$

$$\frac{dV}{dt}\Big|_{\text{That instant}} = \left(\frac{1}{3}\pi(50)^2 4 + \frac{2}{3}\pi(50) 3\, 200\right) \frac{in^3}{min}$$