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9. An ant walks along the curve $\vec{r}(t) = t \cos t \vec{i} + t \sin t \vec{j} + t \vec{k}$, for $0 \leq t$.
Where does the ant touch $x^2 + y^2 + z^2 = 1$?

For ant hits the sphere where $\underbrace{t^2 \cos^2 t + t^2 \sin^2 t}_{t^2} + t^2 = 1 \quad 2t^2 = 1 \quad t^2 = \frac{1}{2}$

$t = \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$

The ant hits the sphere at $\left(\frac{1}{\sqrt{2}} \cos\left(\frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}} \sin\left(\frac{1}{\sqrt{2}}\right), \frac{1}{\sqrt{2}} \right)$.

10. Find the length of the curve $\vec{r}(t) = \frac{t^3}{3} \vec{i} + \frac{t^2}{2} \vec{j}$ for $0 \leq t \leq 1$.

$$\int_0^1 \|\vec{r}'(t)\| dt = \int_0^1 \|t^2 \vec{i} + t \vec{j}\| dt = \int_0^1 \sqrt{t^4 + t^2} dt = \int_0^1 t \sqrt{t^2 + 1} dt$$

$$= \frac{1}{3} \left(2^{\frac{3}{2}} - 1 \right)$$