

5. What are the equations of the line tangent to the curve which is parameterized by $\vec{r}(t) = (3t^3 + 2)\vec{i} + 6t^2\vec{j} + (4t^3 + 2t)\vec{k}$ at $t = 1$?

$$\vec{r}(1) = 5\vec{i} + 6\vec{j} + 6\vec{k} \text{ so } r = (5, 6, 6)$$

$$\vec{r}'(t) = 9t^2\vec{i} + 12t\vec{j} + (12t^2 + 2)\vec{k}$$

$$\vec{r}'(1) = 9\vec{i} + 12\vec{j} + 14\vec{k}$$

The tangent line is

$$x = 5 + 9t$$

$$y = 6 + 12t$$

$$z = 6 + 14t$$

6. Find the equation of the plane tangent to the surface $z = x^2 + 3y^3$ at the point where $x = 2$ and $y = -2$.

$$z = 4 + 3(-8) = 4 - 24 = -20$$

$$\text{So we have } \nabla f(x, y, z) = 0 = 2x\vec{i} + 9y^2\vec{j} - \vec{k}$$

$$\nabla f(2, -2, -20) = 4\vec{i} + 36\vec{j} - \vec{k}$$

$$\text{Hence } 4(x-2) + 36(y+2) - (z+20) = 0$$

$$4(x-2) + 36(y+2) - (z+20) = 0$$