Math 241, Final Exam, Spring, 2023

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 100 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

No Calculators, Cell phones, computers, notes, etc.

(1) Find the equation of the plane that contains the points $(0,1,2)$, $(-1,2,3)$, and $(-4,-1,2)$. **DEMONSTRATE that your answer is correct.**

(2) Express $\overrightarrow{v} = 3\overrightarrow{i} + 5\overrightarrow{j} + \overrightarrow{k}$ as the sum of a vector parallel to $\overrightarrow{w} = \overrightarrow{i} + 2\overrightarrow{j} - \overrightarrow{k}$ and a vector perpendicular to $\overrightarrow{w}$. **DEMONSTRATE that your answer is correct.**

(3) Find the maximum of $f = 49 - x^2 - y^2$ on the line $x + 3y = 10$.

(4) Find the volume between $z = 2 - x^2 - y^2$ and $z = x^2 + y^2 - 2$. (You must draw a meaningful picture.)

(5) Compute $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$.

(6) Find the absolute extreme points of $f(x,y) = 2 + 2x + 4y - x^2 - y^2$ on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, and $y = 9 - x$.

(7) Find the directional derivative of $f(x,y,z) = x^3 - xy^2 - z$ at the point $P = (1,1,0)$, in the direction of $\overrightarrow{v} = 2\overrightarrow{i} - 3\overrightarrow{j} + 6\overrightarrow{k}$.

(8) Find the volume of the solid above the upper part of $x^2 + y^2 = 3z^2$ and below $x^2 + y^2 + z^2 = 1$.

(9) Find the local maxima, local minima, and saddle points of $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$.

(10) Find the area of the region bounded by $y + x^2 = 2$ and $y + x = 0$. (You must draw a meaningful picture.)