Math 241, Final Exam, Fall, 2024

YOU SHOULD KEEP THIS PIECE OF PAPER. Write everything on the **blank paper provided**. Return the problems **IN ORDER** (use as much paper as necessary), use **ONLY ONE SIDE** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. **Fold your exam in half** before you turn it in.

The exam is worth 100 points. There are 10 problems; each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

No Calculators, Cell phones, computers, notes, etc.

(1) Find the equation of the plane that contains the points $P = (1, 1, 1)$, $Q = (-3, 1, -1)$, and $R = (-2, 3, 1)$. **Please make sure that your answer is correct.**

(2) Express $\overrightarrow{v} = \overrightarrow{i} + 7\overrightarrow{j}$ as the sum of a vector parallel to $\overrightarrow{b} = \overrightarrow{i} + 2\overrightarrow{j}$ and a vector perpendicular to $\overrightarrow{b}$. **Please make sure that your answer is correct.**

(3) Consider the function $f(x, y) = x - y^2$ and the point $P = (2, 2)$.

(a) Find the gradient of f at P.

(b) Find the directional derivative of f in the direction of $\overrightarrow{v} = 3\overrightarrow{i} - \overrightarrow{j}$ at P.

(c) Draw the level set of f that contains P.

(d) Draw the gradient of f at P; put the tail of the gradient on P.

(4) Find the length of the curve $\overrightarrow{r}(t) = \cos 2t\,\overrightarrow{i} + \sin 2t\,\overrightarrow{j} + t\,\overrightarrow{k}$, for $0 \leq t \leq \pi$.

(5) Graph, name, and describe the set of all points in 3-space which satisfy the equation $z^2 - x^2 - y^2 = 1$.

(6) Find the absolute maximum and minimum values of

$$f(x, y) = 2 + 2x + 4y - x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, and $y = 9 - x$.

PLEASE TURN OVER.

(7) Find the volume of the solid between $z = 4 - x^2 - y^2$ and $z = x^2 + y^2 - 4$.

(8) Compute $\int_0^1 \int_y^1 e^{x^2}\, dx\, dy$.

(9) Find the area of the region bounded by $x = -y^2$ and $y = x + 2$. (You must draw a meaningful picture.)

(10) Find parametric equations for the line tangent to the curve

$$\vec{\boldsymbol{r}}(t) = t^2 \vec{\boldsymbol{i}} + t^3 \vec{\boldsymbol{j}}$$

at the point $(4, 8)$.