

Please PRINT your name _____

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

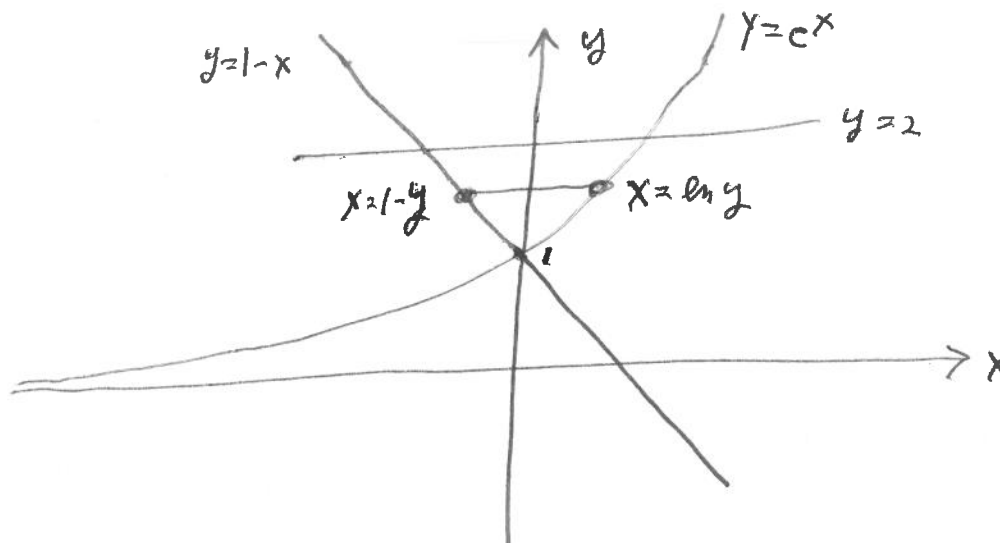
Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will return your quiz when I next see you.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 7, November 13, 2024

Find the area of the region bounded by $y = 1 - x$, $y = 2$, and $y = e^x$. (You must draw the region.)

Answer:



The picture shows that we should use horizontal lines to fill the region and that for each fixed y , with $1 \leq y \leq 2$, x goes from $x = 1 - y$ to $x = \ln y$. The area is

$$\begin{aligned} & \int_1^2 \int_{1-y}^{\ln y} dx dy \\ &= \int_1^2 x \Big|_{1-y}^{\ln y} dy \\ &= \int_1^2 \ln y - (1 - y) dy \end{aligned}$$

Recall that one uses integration by parts $\int u dv = uv - \int v du$ to compute $\int \ln y dy$. Take $u = \ln y$ and $dv = dy$. Compute $du = \frac{1}{y} dy$ and $v = y$. So,
 $\int \ln y dy = \int u dv = uv - \int v du = y \ln y - \int dy = y \ln y - y$.

$$\begin{aligned} &= (y \ln y - y - y + \frac{y^2}{2}) \Big|_1^2 \\ &= (2 \ln 2 - 2 - 2 + \frac{2^2}{2}) - (1 \ln 1 - 1 - 1 + \frac{1^2}{2}) \\ &= (2 \ln 2 - 4 + 2) - (-2 + \frac{1}{2}) = \boxed{2 \ln 2 - \frac{1}{2}} \end{aligned}$$