

PJS

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14. Compute $\int_C 2x \, dx - y^3 \, dy$ where C is the line segment from $(1, 2)$ to $(5, 4)$.

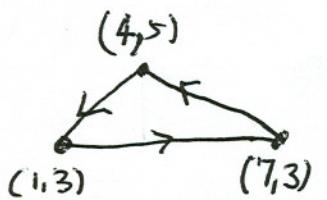
$$\begin{array}{l}
 \text{Diagram of line segment from } (1, 2) \text{ to } (5, 4) \\
 \text{slope } \frac{1}{2} \\
 y - 2 = \frac{1}{2}(x - 1) \\
 2y - 4 = x - 1
 \end{array}$$

$$= \left[25 - \frac{(4)^4}{4} - 1 + \frac{(2)^4}{4} \right]$$

$$\begin{array}{l}
 \text{Let } x = t \\
 y = \frac{t}{2} + \frac{3}{2}
 \end{array}$$

$$1 \leq t \leq 5$$

$$\begin{aligned}
 \text{ans} &= \int_1^5 \left[2t - \left(\frac{t}{2} + \frac{3}{2} \right)^3 \frac{1}{2} \right] dt \\
 &= \left. t^2 - \frac{\left(\frac{t}{2} + \frac{3}{2} \right)^4}{4} \right]_1^5
 \end{aligned}$$

15. Compute $\int_C 2y \, dx - 5x \, dy$ where C consists of three line segments. The first line segment for C starts at $(1, 3)$ and goes to $(7, 3)$; the second segment is from $(7, 3)$ to $(4, 5)$; and the third segment is from $(4, 5)$ to $(1, 3)$.

use Green's Thm $\text{ans} = \iint_{\text{region}} N_x - M_y \, dA$

$$\begin{aligned}
 &= \iint_{\text{region}} (5 - 2) \, dA = -7 \text{ area of region} \\
 &= -7 \left(\frac{1}{2} \right) \text{ base } 6 \text{ height } 2 = -7 \left(\frac{1}{2} \right) (6) 2 = \boxed{-42}
 \end{aligned}$$

