12. Find the absolute extreme points of $f(x,y) = x^2 - y^2 - y$ on $\{(x,y) \mid -2 \le x \le 2,\ -4 \le y \le 2\}$.

interval critical pts:

$f_x = 2x$

$f_y = -2y - 1$

$f_x = 0$ and $f_y = 0$ at $(0, -\frac{1}{2})$

The boundary has four pieces

② $y = 2$ $-2 \le x \le 2$

③ $x = -2$ $-4 \le y \le 2$

① $x = 2$ $-4 \le y \le 2$

④ $y = -4$ $-2 \le x \le 2$

on piece 1 $f(y) = 4 - y^2 - y$

$f'(y) = 2y - 1$

$f' = 0$ at $(2, -\frac{1}{2})$

piece 2 $f(x) = x^2 - 6$

$f'(x) = 2x$

$f' = 0$ at $(0, 2)$

on piece 3

$f(y) = 4 - y^2 - y$

$f'(y) = 0$ when

$(x, y) = (-2, -\frac{1}{2})$

on piece 4

$f(x) = x^2 - 16 + 4$

$f' = 2x$ $f' = 0$ when

$(x, y) = (0, -4)$

I also must consider the corners

$f(0, -\frac{1}{2}) = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$

$f(2, -\frac{1}{2}) = 4 + \frac{1}{4}$

$f(0, 2) = -4 - 2 = -6$

$f(-4, -\frac{1}{2}) = 4 - \frac{1}{4} + \frac{1}{2} = 4 + \frac{1}{4}$

$f(0, -4) = -16 + 4 = -12$

$f(2, 2) = 4 - 4 - 2 = -2$

$f(-2, 2) = 4 - 4 - 2 = -2$

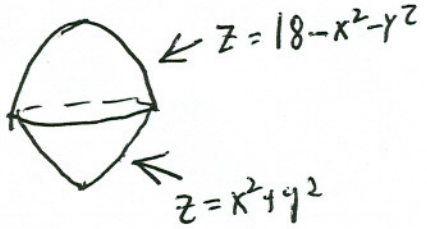
$f(2, -4) = 4 - 16 + 4 = -8$

$f(-2, -4) = 4 - 16 + 4 = -8$

min $(0, -4, -12)$

max $(2, -\frac{1}{2}, \frac{1}{4})$, $(-2, -\frac{1}{2}, \frac{1}{4})$

13. Find the volume of the solid which is bounded by $z = 18 - x^2 - y^2$ and $z = x^2 + y^2$.



intersection is $x^2 + y^2 = 18 - x^2 - y^2$

$2(x^2 + y^2) = 18$

$x^2 + y^2 = 9$

$$Vol = \int_0^{2\pi} \int_0^3 \left[(18 - r^2) - r^2\right] r\, dr\, d\theta$$

$$= 2\pi \int_0^3 18r - 2r^3\, dr$$

$$= 2\pi \left(9r^2 - \frac{2r^4}{4}\right)\Big]_0^3$$

$$= \boxed{2\pi \left[81 - \frac{81}{2}\right]}$$

The level sets of f look like