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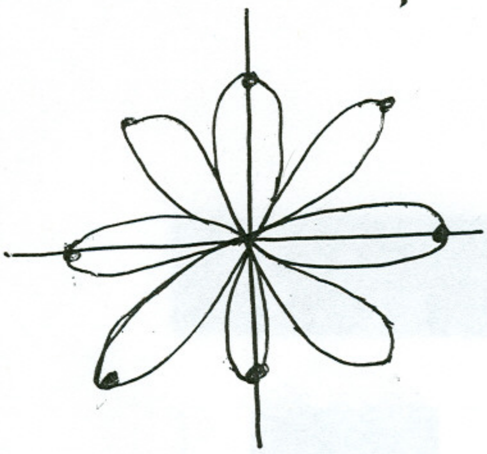
9. Find the area inside one loop of $r = 2\cos 4\theta$.

r	θ
2	0
0	$\frac{\pi}{8}$
-2	$\frac{2\pi}{8}$
0	$\frac{3\pi}{8}$
2	$\frac{4\pi}{8}$
0	$\frac{5\pi}{8}$
-2	$\frac{6\pi}{8}$
0	$\frac{7\pi}{8}$
2	$\frac{8\pi}{8}$
0	$\frac{9\pi}{8}$

r	θ
-2	$\frac{10\pi}{8}$
0	$\frac{11\pi}{8}$
2	$\frac{12\pi}{8}$
0	$\frac{13\pi}{8}$
-2	$\frac{14\pi}{8}$
0	$\frac{15\pi}{8}$
2	$\frac{16\pi}{8}$



$$\text{area} = \int_{-\pi/8}^{\pi/8}\int_0^{2\cos 4\theta} r\,dr\,d\theta$$

$$= \int_{-\pi/8}^{\pi/8} \frac{1}{2}r^2\Big]_0^{2\cos 4\theta}\,d\theta$$

$$= \int_{-\pi/8}^{\pi/8} 2\cos^2 4\theta\,d\theta$$

$$= \int_{-\pi/8}^{\pi/8} 1+\cos 8\theta\,d\theta$$

$$= \theta + \frac{\sin 8\theta}{8}\Big]_{-\pi/8}^{\pi/8} = \frac{\pi}{4}$$

10. Find the point on $(x+13)^2 + (y-6)^2 + (z-1)^2 = 9$ which is closest to $x + 2y + 3z = 100$.

$\pm 3 \frac{|\vec{v}|}{\|\vec{v}\|}$ $\pm 3\,(x+2y+3z)$

The distance from (x,y,z) to $x+2y+3z=100$ is $\dfrac{|x+2y+3z-100|}{\sqrt{1+4+9}}$

So I will minimize $f = x+2y+3z-100$ subject to the constraint $(x+13)^2+(y-6)^2+(z-1)^2=9$

I'll use Lagrange multipliers. I want all points on $(x+13)^2+(y-6)^2+(z-1)^2=9$ where $2(x+13)\vec{i} + 2(y-6)\vec{j} + 2(z-1)\vec{k} = \lambda(\vec{i}+2\vec{j}+3\vec{k})$

so $x = \frac{\lambda}{2} - 13$, $y = \lambda + 6$, $z = \frac{3\lambda}{2} + 1$

plug this into the sphere eq to get

$$\left(\frac{\lambda}{2}\right)^2 + \lambda^2 + \left(\frac{3\lambda}{2}\right)^2 = 9$$

$$\frac{\lambda^2}{4}[1+4+9] = 9$$

$$\lambda = \pm\frac{6}{\sqrt{14}}$$

So the answer is either

$$\left(\frac{3}{\sqrt{14}} - 13,\ \frac{6}{\sqrt{14}} + 6,\ \frac{9}{\sqrt{14}} + 1\right) = P_1$$

or $$\left(\frac{-3}{\sqrt{14}} - 13,\ \frac{-6}{\sqrt{14}} + 6,\ \frac{-9}{\sqrt{14}} + 1\right) = P_2$$

$f(P_1) = \frac{36}{\sqrt{14}} - 98$ $\quad f(P_2) = \frac{-36}{\sqrt{14}} - 98$

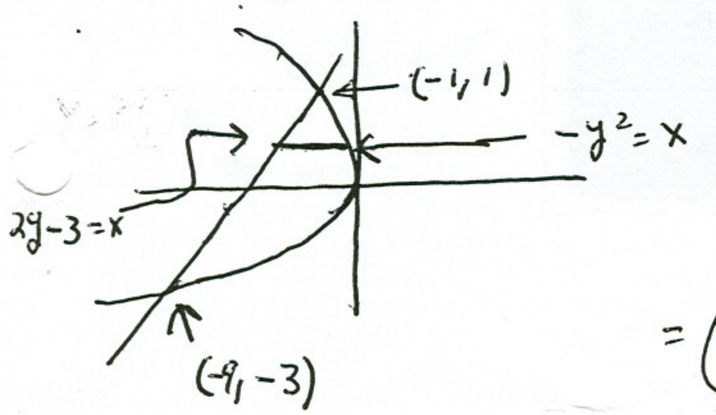
Thus P_1 is the closest point

11. Find the area of the region which is bounded by $x + y^2 = 0$ and $2y - x = 3$.

intersection $2y - 3 + y^2 = 0$

$(y+3)(y-1) = 0$

$y = -3, 1$



$$\text{area} = \int_{-3}^{+1}\int_{2y-3}^{-y^2} dx\,dy$$

$$= \int_{-3}^{+1} (-y^2 - 2y + 3)\,dy$$

$$= -\frac{y^3}{3} - y^2 + 3y\Big]_{-3}^{+1}$$

$$= -\frac{1}{3} - 1 + 3 - (-9 - 9 + 9)$$