

6. An ant walks along the curve $\vec{r}(t) = t \cos t \vec{i} + t \sin t \vec{j} + t \vec{k}$, for $0 \leq t$.
Where does the ant touch $x^2 + y^2 + z^2 = 1$?

$$t^2 \cos^2 t + t^2 \sin^2 t + t^2 = 1$$

$$2t^2 = 1$$

$$t = \pm \frac{1}{\sqrt{2}} \text{ because } t \geq 0.$$

the ant touches the sphere at $\left(\frac{\cos(\frac{1}{\sqrt{2}})}{\sqrt{2}}, \frac{\sin(\frac{1}{\sqrt{2}})}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$

7. Let $f(x, y) = e^{x \sin(xy)}$. Find $\vec{\nabla} f$.

$$\vec{\nabla} f = e^{x \sin(xy)} \left[xy \cos(xy) + \sin(xy) \right] \vec{i} + x^2 \cos(xy) \vec{j}$$

8. Let $f(x, y) = \frac{y^2}{y^2 + x^4}$. Calculate the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ along $y = 2x^2$.

$$\lim_{x \rightarrow 0} \frac{(2x^2)^2}{(2x^2)^2 + x^4} = \lim_{x \rightarrow 0} \frac{4x^4}{5x^4} = \left(\frac{4}{5} \right)$$