MATH 241, FALL 2001, EXAM 4

PRINT Your Name:______________________________

There are 8 problems on 5 pages. Problems 1 through 7 are each worth 10 points. Problem 8 has three parts; each part is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS! Check your answer whenever possible.** If you want to pick up your exam before Tuesday, write a short note to that effect on the top of this page and I will leave your exam outside my office door, before I go home tonight.

1. If $f(x,y) = xe^{xy}$, then find $\overrightarrow{\nabla} f$.

2. Find the directional derivative of $f(x,y) = x^2y$ at $(1,2)$ in the direction $\overrightarrow{\boldsymbol{u}} = \frac{3}{5}\overrightarrow{\boldsymbol{i}} - \frac{4}{5}\overrightarrow{\boldsymbol{j}}$.

3.
   (a) Find $\lim\limits_{\substack{(x,y)\to(0,0) \\ \text{along } y=3x}} \frac{x^3y}{x^6+2y^2}$.

   (b) Find $\lim\limits_{\substack{(x,y)\to(0,0) \\ \text{along } y=x^3}} \frac{x^3y}{x^6+2y^2}$.

4. Find the slope of the line tangent to the curve of intersection of the surface $36z = 4x^2 + 9y^2$ and the plane $x = 3$ at the point $(3,2,2)$.

5. Find the equation of the **plane tangent** to $z^2 = x^2 + y^2$ at $(3,4,5)$.

6. Find the equation of the **line perpendicular** to $z^2 = x^2 + y^2$ at $(3,4,5)$.

7. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 80 inches and is increasing at 5 inches per minute and the radius is 50 inches and is increasing at 2 inches per minute. How fast is the volume increasing at that instant? (The volume of a cone is $V = (1/3)\pi r^2 h$.)

8. **Each part is worth 10 points.** The temperature of a plate at the point (x,y) is $T(x,y) = xy$.
   (a) Draw and label the level sets $T = 0$, $T = 1$, $T = -1$, $T = 2$, and $T = -2$.
   (b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point $(2,-1)$. Draw the path of the particle.
   (c) Find the equation which gives the path of the particle of part (b).