Math 241, Fall 2000, Exam 4

There are 10 problems on 6 pages. Each problem is worth 10 points. SHOW your work. $\boxed{\mathit{CIRCLE}}$ your answer. **NO CALCULATORS!**

1. Find $\displaystyle\int_0^1 \int_0^{3x} x^2 \, dy \, dx$.

2. Find the volume of the solid in the first octant which is bounded by $y = x^2$, $x = 0$, $z = 0$, and $y + z = 1$.

3. Let R be the region $R = \{(x, y) \mid 2 \le x \le 8, \text{ and } 2 \le y \le 6\}$. Let P be the partition of R into six equal squares by the lines $x = 4$, $x = 6$, and $y = 4$. Approximate $\displaystyle\iint_R (12 - x - y) \, dA$ by calculating the corresponding Riemann sum $\sum_{k=1}^{6} f(\bar{x}_k, \bar{y}_k) \Delta A_k$, where $(\bar{x}_k, \bar{y}_k)$ is the center of the k^{th} box.

4. Identify all local maximum points, all local maximum points, and all saddle points of $f(x, y) = xy^2 - 6x^2 - 6xy$.

5. Where does the line normal to $x^2 + y^2 + 2z^2 = 6$ at $(1, 2, 1)$ intersect $2x + 3y + z = 49$?

6. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 100 inches and is increasing at 3 inches per minute and the radius is 40 inches and is increasing at 2 inches per minute. How fast is the volume increasing at that instant? (The volume of a cone is $V = (1/3)\pi r^2 h$.)