Math 241, Fall 2000, Exam 3

PRINT Your Name:____________________________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1. Graph and name $y = x^2 + z^2$ in $3-$ space.

2. Graph and name $y^2 = x^2 + z^2$ in $3-$ space.

3. Draw and label the level sets of levels $k = -1$, $k = 0$, and $k = 1$ for the function $f(x, y) = y^2 - x^2$.

4. Let $f(x, y) = xe^{\frac{y}{x}}$. Find f_x and f_y.

5. (15 points) Let $f(x, y) = \dfrac{xy + y^4}{x^2 + y^2}$.
   (a) Calculate the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ along $x = 0$.
   (b) Calculate the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ along $y = x$.
   (c) What is $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$?

6. The temperature of a metal plate at (x, y) is e^{-x-3y} degrees. A bug is walking northeast at a rate of $\sqrt{8}$ feet per minute (that is, $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = 2$). From the bug's point of view, how is the temperature changing with time as it crosses the origin?

7. Find the slope of the line tangent to the curve which is the intersection of the surface $36z = 4x^2 + 9y^2$ and the plane $x = 3$ at the point $(3, 2, 2)$.

8. A fly is crawling along a wire helix so that its position vector is

$$\overrightarrow{\boldsymbol{r}} = 6\cos \pi t\, \overrightarrow{\boldsymbol{i}} + 6 \sin \pi t\, \overrightarrow{\boldsymbol{j}} + 2t\, \overrightarrow{\boldsymbol{k}}$$

   for $0 \leq t$. At what point will the fly hit the sphere $x^2 + y^2 + z^2 = 100$ and how far did it travel in getting there (assuming it started when $t = 0$)?

9. Find the equations of the line through $(4, 0, 6)$ and perpendicular to the plane $x - 5y + 2z = 10$.

10. Find the equation of the plane that contains the parallel lines

$$\begin{cases} x = -2 + 2t \\ y = 1 + 4t \\ z = 2 - t \end{cases} \quad \text{and} \quad \begin{cases} x = 2 - 2t \\ y = 3 - 4t \\ z = 1 + t \end{cases}$$