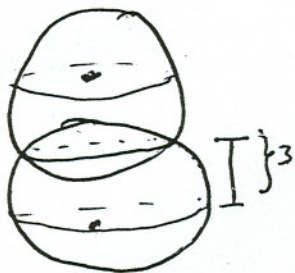




7. Find the volume of the intersection of $x^2 + y^2 + (z-6)^2 \leq 16$ and $x^2 + y^2 + z^2 \leq 16$.



$$2 \int_0^{2\pi} \int_0^{\sqrt{7}} \int_3^{\sqrt{16-r^2}} r \, dz \, dr \, d\theta$$

the intersection is $z=3$ and $r^2 + z^2 = 16$
so $r^2 = 7$ $r = \sqrt{7}$

$$= 4\pi \int_0^{\sqrt{7}} r \sqrt{16-r^2} - 3r \, dr$$

$$= 4\pi \left[\frac{2}{3} (16-r^2)^{3/2} - \frac{3r^2}{2} \right]_0^{\sqrt{7}}$$

$$= 4\pi \left(-\frac{1}{3}(27) - \frac{21}{2} + \frac{64}{3} \right)$$

$$= \frac{4\pi}{6} (2(37) - 3(21))$$

$$= \frac{22\pi}{3}$$