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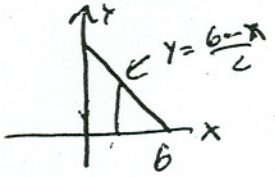
5. Find the volume of the solid which is bounded by $x = 0$, $y = 0$, $z = 0$, and $x + 2y + 3z = 6$.

$$\int_0^6 \int_0^{\frac{6-x}{2}} \int_0^{\frac{6-x-2y}{3}} dz\,dy\,dx = \int_0^6 \int_0^{\frac{6-x}{2}} 2 - \frac{x}{3} - \frac{2y}{3}\,dy\,dx$$



$$= \int_0^6 2y - \frac{xy}{3} - \frac{y^2}{3}\Big]_0^{\frac{6-x}{2}}\,dx$$

$$= \int_0^6 6 - x - \frac{x(6-x)}{6} - \frac{36 - 12x + x^2}{12}\,dx = \frac{1}{12}\int_0^6 72 - 12x - 12x + 2x^2 - 36 + 12x - x^2\,dx$$

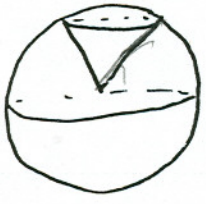
$$= \frac{1}{12}\int_0^6 36 - 12x + x^2\,dx = \frac{1}{12}\left(36x - 6x^2 + \frac{x^3}{3}\right)\Big]_0^6 = \frac{1}{12}\left(36\cdot 6 - 6\cdot 36 + 36\cdot 2\right)$$

$$= 6$$

6. Find the volume of the solid which is bounded by $z = \sqrt{9 - x^2 - y^2}$ and $z = \sqrt{x^2 + y^2}$.

Hemi-sphere

cone



$$Vol = \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^3 \rho^2 \sin\varphi\,d\rho\,d\varphi\,d\theta = 2\pi \qquad \frac{\rho^3}{3}\Big]_0^3 \quad (-\cos\varphi)\Big]_0^{\frac{\pi}{4}}$$

$$= 2\pi \cdot 9\left(1 - \frac{\sqrt{2}}{2}\right)$$