

3. Find the volume of the solid which is bounded by $z = 9 - x^2 - y^2$ and $z = 0$.

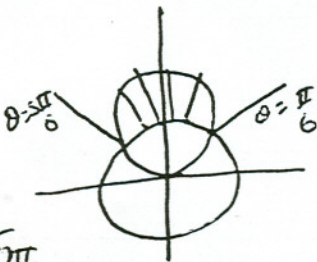


Circle
of radius 3

$$\int_0^{2\pi} \int_0^3 \int_0^{9-r^2} r \, dz \, dr \, d\theta = 2\pi \int_0^3 r(9-r^2) \, dr = 2\pi \left[\frac{9r^2}{2} - \frac{r^4}{4} \right]_0^3$$

$$= \left(\frac{81}{2} - \frac{81}{4} \right) 2\pi = \left(\frac{81}{4} \right) 2\pi = \boxed{\frac{81\pi}{2}}$$

4. Find the area inside $r = 4 \sin \theta$ and outside $r = 2$.



intersection $2 = 4 \sin \theta$

$$\frac{1}{2} = \sin \theta$$

$$\frac{\pi}{6}, \frac{5\pi}{6} = \theta$$

$$\begin{aligned} \int_{\pi/6}^{5\pi/6} \int_2^{4\sin\theta} r \, dr \, d\theta &= \int_{\pi/6}^{5\pi/6} \left[\frac{r^2}{2} \right]_2^{4\sin\theta} d\theta = \int_{\pi/6}^{5\pi/6} \left(\frac{16 \sin^2 \theta}{2} - \frac{4}{2} \right) d\theta \\ &= \int_{\pi/6}^{5\pi/6} \left(8 \frac{1 - \cos 2\theta}{2} - 2 \right) d\theta = \left[2\theta - 2 \sin 2\theta \right]_{\pi/6}^{5\pi/6} \end{aligned}$$

$$= \frac{10\pi}{6} + 2\frac{\sqrt{3}}{2} - \left(\frac{2\pi}{6} - 2\frac{\sqrt{3}}{2} \right) = \boxed{\frac{4\pi}{3} + 4\frac{\sqrt{3}}{2}}$$