Spring 1998 Exam 4 241

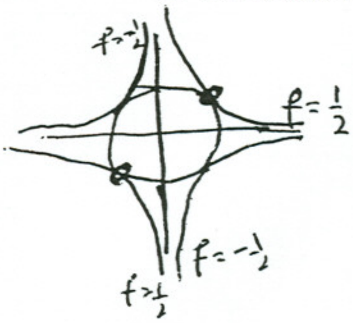
PRINT Your Name:________________________

There are 7 problems on 4 pages. Problems 1 and 2 are worth 15 points each. Each of the other problems is worth 14 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

84

100

1. Find the maximum of $f(x,y) = xy$ on $x^2 + y^2 = 1$.



I use Lagrange Multipliers

$\vec{\nabla} f = \lambda \vec{\nabla}(x^2+y^2)$

$y\vec{i} + x\vec{j} = \lambda(2x\vec{i} + 2y\vec{j})$

so $\begin{cases} y = \lambda 2x \\ x = \lambda 2y \\ x^2+y^2=1 \end{cases}$

so $x = \lambda 2(\lambda 2 x)$

so either x and y are both 0 which is not on the circle

or $1 = 4\lambda^2$

so $\lambda = \pm \frac{1}{2}$

$\therefore y = \pm x$

So the max and the min of f on $x^2+y^2=1$ occur at one of

$\frac{1}{2} = f\left(\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}\right)$ Max

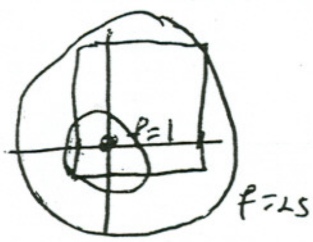
$-\frac{1}{2} = f\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ min

$-\frac{1}{2} = f\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ min

$\frac{1}{2} = f\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ Max

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right), \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{1}{2}\right)$

2. Find the absolute extreme points of $f(x,y) = x^2 + y^2$ on $\{(x,y) \mid -1 \le x \le 3,\ -1 \le y \le 4\}$.



interior critical pts

$f_x = 2x \quad f_y = 2y$

both partials are 0 at $(0,0)$

along $x=-1$ $f = 1+y^2$ $f' = 0$ at $(-1,0)$

along $y=-1$ $f = x^2+1$ $f' = 0$ at $(0,-1)$

along $x=3$ $f = 9+y^2$ $f' = 0$ at $(3,0)$

along $y=4$ $f = x^2+16$ $f' = 0$ at $(0,4)$

corners are $(-1,-1)$ $(-1,4)$ $(3,-1)$ $(3,4)$

$f(0,0) = 0$

$f(-1,0) = 1$

$f(0,-1) = 1$

$f(3,0) = 9$

$f(0,4) = 16$

$f(-1,-1) = 2$

$f(-1,4) = 17$

$f(3,-1) = 10$

$f(3,4) = 25$

Max $(3,4,25)$

min $(0,0,0)$