123

~~108~~

3. Identify all local maximum points, all local maximum points, and all saddle points of $f(x,y) = 2x^4 - x^2 + 3y^2$.

$f_x = 8x^3 - 2x = 2x(4x^2-1) = 2x(2x-1)(2x+1)$

$f_y = 6y$

$f_x = 0$ and $f_y = 0$ at $(0,0)$, $(\frac{1}{2},0)$, $(-\frac{1}{2},0)$

$f_{xx} = 24x^2 - 2$

$f_{xy} = 0$

$f_{yy} = 6$

$D = f_{xx}f_{yy} - (f_{xy})^2 = 6(24x^2-2)$

at $(0,0)$ $D = -12 < 0$

So $(0,0,0)$ is a saddle point.

at $(\frac{1}{2},0)$ $D = 6(6-2) > 0$ $f_{xx} > 0$

So $(\frac{1}{2}, 0, f(\frac{1}{2},0))$ is a local min

at $(-\frac{1}{2},0)$ $D = 6(4) > 0$ $f_{xx} = 4 > 0$

$(-\frac{1}{2}, 0, f(-\frac{1}{2},0))$ is a local min

This is 15.8/3

4. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 60 inches and is increasing at 4 inches per minute and the radius is 30 inches and is increasing at 3 inches per minute. How fast is the volume increasing at that instant? (The volume of a cone is $V = (1/3)\pi r^2 h$.)

$$\frac{dV}{dt} = \frac{1}{3}\pi\, 2r\frac{dr}{dt}h + \frac{1}{3}\pi r^2 \frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{\text{at that instant}} = \left(\frac{1}{3}\pi\, 2(30)3\cdot 60 + \frac{1}{3}\pi(30)^2 4\right)\frac{in^3}{min}$$

$r = 30$ $\frac{dr}{dt} = 3$ $h = 60$ $r = 30$ $\frac{dh}{dt} = 4$