9. Find the volume of the solid bounded by the xy plane and $z = 9 - x^2 - y^2$.

$$Vol = \int_0^{2\pi}\int_0^{3} (9-r^2)r\,dr\,d\theta = \int_0^{2\pi} \frac{9r^2}{2} - \frac{r^4}{4}\Big]_0^3 d\theta = \left(\frac{81}{2} - \frac{81}{4}\right)\theta\Big]_0^{2\pi}$$

$$= \frac{81}{4}(2\pi)$$

$z = \sqrt{12 - r^2}$

10. Find the volume of the solid whose lower bound is $x^2 + y^2 = z$ and whose upper bound is $x^2 + y^2 + z^2 = 12$.

← the flattest cross section is $z + z^2 = 12$

$z^2 + z - 12 = 0$

$(z+4)(z-3) = 0$

i.e. $z = -4$ or $z = 3$

so $z = 3$

but $z = r^2$ so $r = \sqrt{3}$

$z = x^2 + y^2$

i.e. $z = r^2$

$$Vol = \iint_{\substack{\text{in} \\ \text{flattest} \\ \text{cross} \\ \text{section}}} \int_{\text{bot}}^{\text{top}} dz\,\underbrace{dx\,dy}_{\text{in}} = \int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{\sqrt{12-r^2}} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{3}} r\sqrt{12-r^2} - r^3\,dr\,d\theta = \int_0^{2\pi} -\frac{1}{2}\left(\frac{2}{3}\right)(12-r^2)^{\frac{3}{2}} - \frac{r^4}{4}\Big]_0^{\sqrt{3}} d\theta$$

$$= -\frac{1}{3}(9)^{\frac{3}{2}} - \frac{9}{4} + \frac{1}{3}\left(12^{\frac{3}{2}}\right)$$

$$= -9 - \frac{9}{4} + 8\sqrt{3}$$