

5. Let $R = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 4\}$. Find $\int_R x^2 dA$.

$$\int_0^{2\pi} \int_1^2 r^2 \cos^2 \theta r dr d\theta = \int_0^{2\pi} \left[\frac{r^4}{4} \cdot \frac{1}{2} (1 + \cos 2\theta) \right]_1^2 d\theta = \frac{15}{8} \left(\theta + \frac{\sin 2\theta}{2} \right) \Big|_0^{2\pi} = \frac{30\pi}{8}$$

6. Find $\int_R e^{x^2+y^2} dA$, where R is the region enclosed by $x^2 + y^2 = 4$.

$$\int_0^{2\pi} \int_0^2 r e^{r^2} dr d\theta = \int_0^{2\pi} \left[\frac{1}{2} e^{r^2} \right]_0^2 d\theta = \frac{1}{2} (e^{16} - 1) \theta \Big|_0^{2\pi} = (e^{16} - 1) \pi$$