

11/7/16 (69)

3. Find $\int_0^{\ln 3} \int_0^1 x y e^{xy^2} dy dx = \int_0^{\ln 3} \left. \frac{1}{2} e^{xy^2} \right|_0^1 dx = \frac{1}{2} \int_0^{\ln 3} e^x - 1 dx = \frac{1}{2} (e^x - x) \Big|_0^{\ln 3}$

$= \frac{1}{2} (3 - \ln 3 - 1) = \frac{1}{2} (2 - \ln 3)$

4. Find the volume of the solid in the first octant bounded by $9z = 36 - 9x^2 - 4y^2$ and the coordinate planes.

$\iint \frac{36 - 9x^2 - 4y^2}{9} dx dy \xrightarrow{\text{iso}} \iint \frac{36 - u^2 - v^2}{9} \left| \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} \right| du dv$

$\left| \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} \right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$u = 3x, v = 2y$

$9x^2 + 4y^2 = 36 \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$

$u^2 + v^2 = 36 \Rightarrow \frac{u^2}{36} + \frac{v^2}{36} = 1$

$= \int_0^{\frac{\pi}{2}} \int_0^6 \left(4 - \frac{v^2}{9} \right) r \frac{1}{6} dr d\theta = \frac{1}{6} \int_0^{\frac{\pi}{2}} \left[4r - \frac{r^3}{9} \right]_0^6 d\theta = \frac{1}{6} \int_0^{\frac{\pi}{2}} \left(24 - \frac{216}{9} \right) d\theta = \frac{1}{6} \int_0^{\frac{\pi}{2}} 6 d\theta = \int_0^{\frac{\pi}{2}} 1 d\theta = \left[\theta \right]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$

$u = r \cos \theta, v = r \sin \theta$

$= \int_0^{\frac{\pi}{2}} \frac{1}{6} [2(36) - 36] d\theta = \left[\theta \right]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$

$\boxed{3\pi}$