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5. Find the absolute extreme points of $f(x,y) = x^2 - 6x + y^2 - 8y + 7$ on $\{(x,y) \mid x^2+y^2 \le 1\}$.

Critical interior Points: $f_x = 2x - 6$
$f_y = 2y - 8$

Both partials are zero only at $(3,4)$ which is not in the domain

Thus the extreme points occur on the boundary which is parametrized by

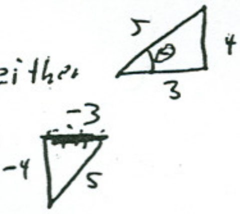
$$\begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases}$$

on the boundary f is given by

$$f(\theta) = \cos^2\theta - 6\cos\theta + \sin^2\theta - 8\sin\theta + 7$$
$$= -6\cos\theta - 8\sin\theta + 8$$
$$f'(\theta) = 6\sin\theta - 8\cos\theta$$
$$f'(\theta) = 0 \text{ when } 6\sin\theta - 8\cos\theta = 0$$

i.e. $\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

So either



$\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{3}{5}$

or

$\sin\theta = -\frac{4}{5}$ $\cos\theta = -\frac{3}{5}$

My critical Pts are

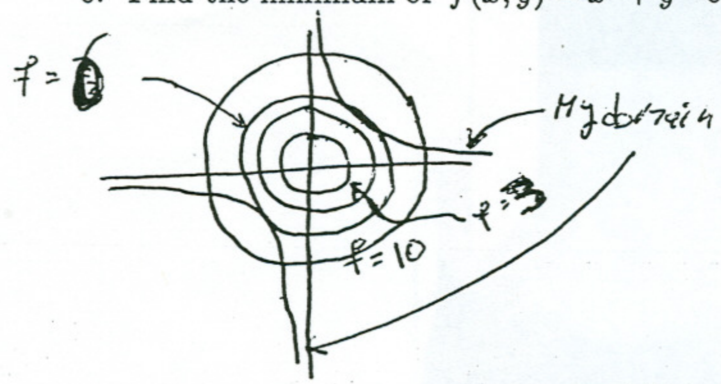
$\left(\frac{3}{5}, \frac{4}{5}\right)$ $\left(-\frac{3}{5}, -\frac{4}{5}\right)$

$$f\left(\tfrac{3}{5},\tfrac{4}{5}\right) = \frac{9}{25} - \frac{18}{5} + \frac{16}{25} - \frac{32}{5} + 7 = 8 - \frac{50}{5} =$$

$$f\left(-\tfrac{3}{5},-\tfrac{4}{5}\right) = \frac{9}{25} + \frac{18}{5} + \frac{16}{25} + \frac{32}{5} + 7 = 8 + 10 = 12$$

The Max is $\left(-\frac{3}{5}, -\frac{4}{5}, 12\right)$
The min is $\left(\frac{3}{5}, \frac{4}{5}, -2\right)$

6. Find the minimum of $f(x,y) = x^2 + y^2$ on $xy = 3$.



The picture shows that the answer is $(\sqrt{3}, \sqrt{3}, 6)$ and $(-\sqrt{3}, -\sqrt{3}, 6)$

LaGrange Multipliers says

$$2x\vec{i} + 2y\vec{j} = \lambda(y\vec{i} + x\vec{j})$$

So we need

$$\begin{cases} 2x = \lambda y \\ 2y = \lambda x \\ xy = 3 \end{cases}$$

so $\begin{cases} \frac{2x}{y} = \lambda \\ 2y = \lambda x \end{cases}$

$$\begin{cases} \frac{2x}{y} = \lambda \\ 2y = \frac{2x^2}{y} \\ xy = 3 \end{cases}$$

$$\begin{cases} \frac{2x}{y} = \lambda \\ y^2 = x^2 \\ xy = 3 \end{cases}$$

so $\begin{cases} \frac{2x}{y} = \lambda \\ y = x \text{ or } y = -x \\ xy = 3 \end{cases}$

so $x = \sqrt{3}$ $y = \sqrt{3}$ $f = 6$

or $x = -\sqrt{3}$ $y = -\sqrt{3}$ $f = 6$ only 1

The min is
$(\sqrt{3}, \sqrt{3}, 6)$ or $(-\sqrt{3}, -\sqrt{3}, 6)$