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3. Let $f(x,y) = 2xy^2$ and let p be the point $p = (1,3)$. Find the directional derivative of f at the point p in the direction of $\vec{v} = 2\vec{i} + 5\vec{j}$.

$D_{\vec{u}} f|_p = \vec{\nabla} f|_p \cdot \vec{u}$ when $\vec{u}$ is a unit vector

$D_{\vec{v}} f|_p = \vec{\nabla} f|_p \cdot \frac{\vec{v}}{\|\vec{v}\|} = (2y^2\vec{i} + 4xy\vec{j})|_p \cdot \frac{(2\vec{i}+5\vec{j})}{\sqrt{29}} = (18\vec{i} + 12\vec{j}) \cdot \frac{(2\vec{i}+5\vec{j})}{\sqrt{29}}$

$= \frac{36+60}{\sqrt{29}}$

$\frac{96}{\sqrt{29}}$

4. Identify all local extreme points and all saddle points of $f(x,y) = 2x^4 - x^2 + 3y^2$.

$f_x = 8x^3 - 2x$ $\qquad f_{xx} = 24x^2 - 2$

$f_y = 6y$ $\qquad f_{xy} = 0$

f_x and $f_y = 0$ when $\qquad f_{yy} = 6$

$y=0$ and $2x(4x^2-1)=0$ $\qquad 8x^3-2x$

so $y=0$ and $x=0$ or $x=\frac{1}{2}$ or $x=-\frac{1}{2}$

at $(0,0)$:

$D = 6(-2) < 0$ so $(0,0,0)$ is a saddle pt

at $(\frac{1}{2},0)$

$D = 6(4) > 0$ $f_{xx} = 4 > 0$ so $(\frac{1}{2}, 0, f(\frac{1}{2},0))$ is a local min

at $(-\frac{1}{2},0)$

$D = 6(4) > 0$ $f_{xx} = 4 > 0$ so $(-\frac{1}{2}, 0, f(-\frac{1}{2},0))$ is a local min