

# Math 241 Fall 1997 exam 4

101



6

PRINT Your Name: \_\_\_\_\_

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. **CIRCLE** your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Find the equations of the line normal to  $z = 3x^2 + 6y^2$  when  $x = 1$  and  $y = -1$ .

$\nabla$  + level sets

My surface is the level set

$$0 = 3x^2 + 6y^2 - z$$

$$\nabla(3x^2 + 6y^2 - z) = 6x\hat{i} + 12y\hat{j} - \hat{k}$$

my pt is  $(1, -1, 9)$

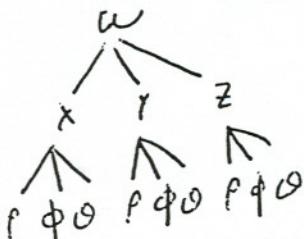
$$\nabla|_{pt} = 6\hat{i} - 12\hat{j} - \hat{k}$$

my line is

$$\frac{x-1}{6} = \frac{y+1}{-12} = \frac{z-9}{-1}$$



2. Suppose that  $w = f(x, y, z)$ , and  $x, y, z$  are written spherical coordinates (that is,  $x = \rho \sin \phi \cos \theta$ ,  $y = \rho \sin \phi \sin \theta$ , and  $z = \rho \cos \phi$ ). Express  $\frac{\partial w}{\partial \phi}$  in terms of  $\frac{\partial w}{\partial x}$ ,  $\frac{\partial w}{\partial y}$ , and  $\frac{\partial w}{\partial z}$ .



$$\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial \phi} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial \phi} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial \phi}$$

$$\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x} \rho \cos \phi \cos \theta + \frac{\partial w}{\partial y} \rho \cos \phi \sin \theta - \frac{\partial w}{\partial z} \rho \sin \phi$$