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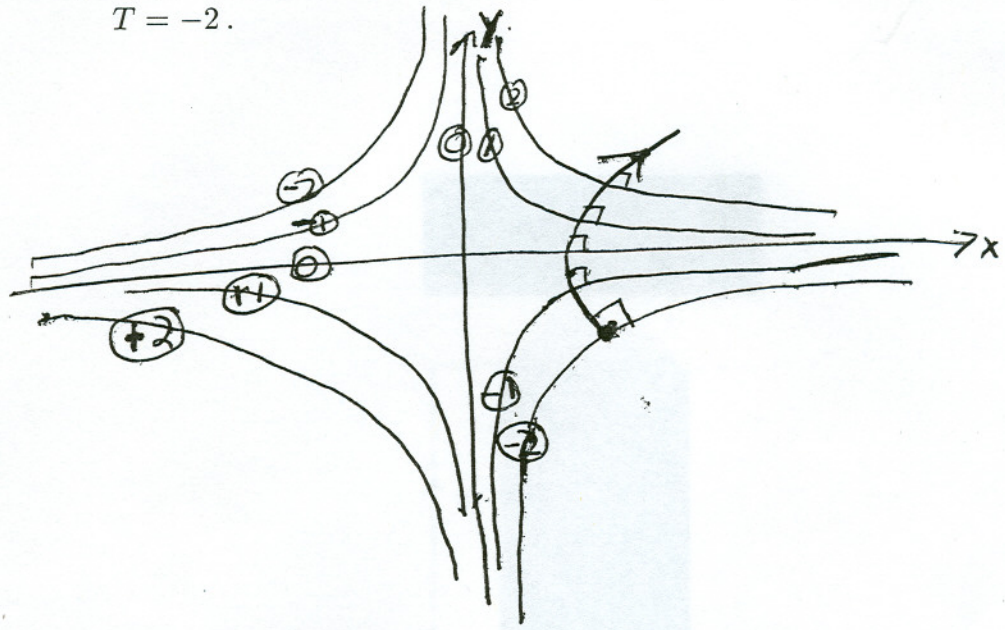
8. **Each part is worth 10 points.** The temperature of a plate at the point (x, y) is $T(x, y) = xy$.

(a) Draw and label the level sets $T = 0$, $T = 1$, $T = -1$, $T = 2$, and $T = -2$.



(b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point $(2, -1)$. Draw the path of the particle.

(c) Find the equation which gives the path of the particle of part (b).

Let $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ be the path of the particle

I know that $\vec{r}\,'(t) \parallel \vec{\nabla} T\big|_t$ at all time. So

$x'(t)\vec{i} + y'(t)\vec{j} = c(t) \cdot \left(y(t)\vec{i} + x(t)\vec{j}\right)$

for some scalar function $c(t)$

So $x'(t) = c(t)\,y(t)$

and $y'(t) = c(t)\,x(t)$

i.e. $\dfrac{x'(t)}{y(t)} = c(t) = \dfrac{y'(t)}{x(t)}$

So $x(t)\,x'(t) = y(t)\,y'(t)$

Integrate with respect to t

$\int x(t)\,x'(t)\,dt = \int y(t)\,y'(t)\,dt$

$\dfrac{(x(t))^2}{2} = \dfrac{(y(t))^2}{2} + C_0$

Multiply through by 2.

Ignore t. Let $K = 2C_0$

$x^2 = y^2 + K$

To find K, plug in $x=4$, $y=-1$

$4 = 1 + K \qquad 3 = K$

$\boxed{x^2 = y^2 + 3}$