120 ~~105~~

10. If the temperature of a plate at the point (x, y) is $T(x, y) = 10 + 2x^2 - y^2$, then find the path a heat-seeking particle (which always moves in the direction of greatest increase of temperature) would follow if it starts at $(4, 2)$.

Let $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ be the position vector of the particle at time t

we are told that $\vec{r}\,'(t) = c(t)\vec{\nabla}T|_t$ for all t and some constant function c

So $\left(x'(t)\vec{i} + y'(t)\vec{j}\right) = c(t)\left(4x(t)\vec{i} - 2y(t)\vec{j}\right)$

so $x'(t) = c(t) \cdot 4x(t)$
$y'(t) = c(t) \cdot -2y(t)$

$$\begin{cases} \dfrac{x'(t)}{4x(t)} = c(t) \\[2ex] \dfrac{y'(t)}{-2y(t)} = c(t) \end{cases}$$

$$\frac{x'(t)}{4x(t)} = \frac{y'(t)}{-2y(t)}$$

$$\int -\frac{1}{2}\frac{x'(t)}{x(t)}\,dt = \int \frac{y'(t)}{y(t)}\,dt$$

$$k - \frac{1}{2}\ln|x(t)| = \ln|y(t)|$$

$$e^k\left(e^{\ln|x(t)|}\right)^{-\frac{1}{2}} = e^{\ln|y(t)|}$$

$$\frac{e^k}{\sqrt{|x(t)|}} = |y(t)|$$

Ignore t

$$\frac{\pm e^k}{\sqrt{x}} = y$$

Plug in $(4,2)$

$$\frac{\pm e^k}{2} = 2 \qquad \pm e^k = 4$$

$$y = \frac{4}{\sqrt{x}}$$