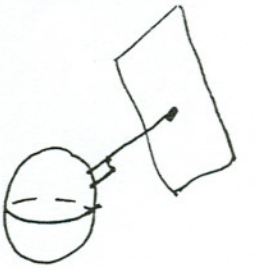
5. Where does the line normal to $x^2 + y^2 + 2z^2 = 6$ at $(1, 2, 1)$ intersect $2x + 3y + z = 49$?

$\vec{\nabla}$'s are $\perp$ level sets.

$\vec{\nabla}(LHS)|_{(121)} = (2x\vec{i} + 2y\vec{j} + 4z\vec{k})|_{(121)} = 2\vec{i} + 4\vec{j} + 4\vec{k}$

The normal line is $x = 1 + 2t$, $y = 2 + 4t$, $z = 1 + 4t$



The normal line hits $2x + 3y + z = 49$ when $2(1+2t) + 3(2+4t) + (1+4t) = 49$

$2 + 6 + 1 + 4t + 12t + 4t = 49$

$20t = 40$

$t = 2$

The normal line hits the plane at $(5, 10, 9)$.

15.7/26

6. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 100 inches and is increasing at 3 inches per minute and the radius is 40 inches and is increasing at 2 inches per minute. How fast is the volume increasing at that instant? (The volume of a cone is $V = (1/3)\pi r^2 h$.)

15.6/20

$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + \frac{2}{3}\pi r h \frac{dr}{dt}$

$\left.\frac{dV}{dt}\right|_{\text{this instant}} = \frac{1}{3}\pi(40)^2 3 + \frac{2}{3}\pi(40)(100)2 \quad \frac{in^3}{min}$