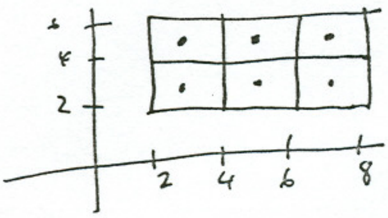
~~10~~ 1/6

3. Let R be the region $R = \{(x, y) \mid 2 \le x \le 8, \text{ and } 2 \le y \le 6\}$. Let P be the partition of R into six equal squares by the lines $x = 4$, $x = 6$, and $y = 4$. Approximate $\iint_R (12 - x - y)\,dA$ by calculating the corresponding Riemann sum $\sum_{k=1}^{6} f(\bar{x}_k, \bar{y}_k)\Delta A_k$, where $(\bar{x}_k, \bar{y}_k)$ is the center of the k^{th} box.



The centers are $(3,5)$ $(5,5)$ $(7,5)$

$(3,3)$ $(5,3)$ $(7,3)$

$$\sum_{k=1}^{6} f(\bar{x}_k, \bar{y}_k)\Delta A_k = 4\left((12-3-5) + (12-5-5) + (12-7-5) + (12-3-3) + (12-5-3) + (12-7-3)\right)$$

each square has area 4

$= 4 \cdot 18 = 72$

Similar to 16.1|9

4. Identify all local maximum points, all local maximum points, and all saddle points of $f(x, y) = xy^2 - 6x^2 - 6xy$.

$f_x = y^2 - 12x - 6y$

$f_y = 2xy - 6x$

$f_x = 0$ and $f_y = 0$ when

$\begin{cases} y^2 - 12x - 6y = 0 \\ 2x(y-3) = 0 \end{cases}$

so either $x = 0$ and $y(y-6) = 0$

or $y = 3$ and $9 - 12x - 18 = 0$ $\quad -9 = 12x$, $-\frac{3}{4} = x$

The critical points are

$(0,0)$, $(0,6)$ $(-\frac{3}{4}, 3)$

$f_{xx} = -12$ $\quad f_{xy} = 2y - 6$ $\quad f_{yy} = 2x$

$D = f_{xx}f_{yy} - (f_{xy})^2 = -24x - (2y-6)^2$

at $(0,0)$. $D = -36$ so $(0,0,0)$ is a saddle pt

at $(0,6)$ $D = -36$ so $(0,6,0)$ is a saddle pt

at $(-\frac{3}{4}, 3)$:

$D = -24(-\frac{3}{4}) - 0^2 = +18 > 0$

$f_{xx} < 0$

$(-\frac{3}{4}, 3, f(-\frac{3}{4}, 3))$ is a local max