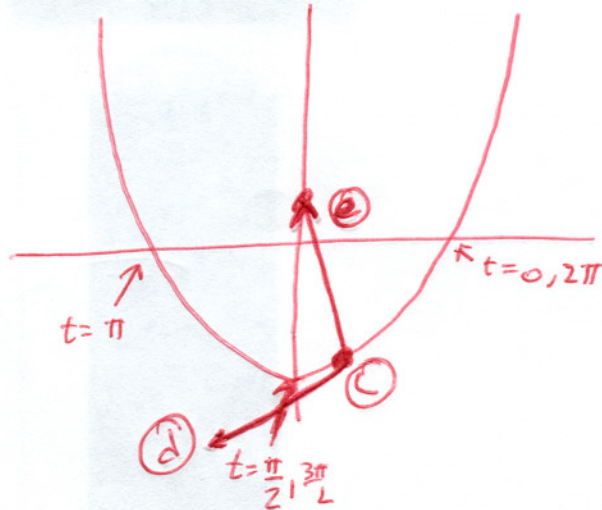10. The position of a moving particle at time t is given by the position vector

$$\vec{r}(t) = 2\cos t\,\vec{i} - 3\sin^2 t\,\vec{j}.$$

(a) Graph the path of the object.
(b) Eliminate the parameter and express the path of the object in cartesian coordinates.
(c) Which point on the curve corresponds to $t = \frac{\pi}{3}$?
(d) Draw $\vec{v}\left(\frac{\pi}{3}\right)$. Point the tail on your answer to (c).
(e) Draw $\vec{a}\left(\frac{\pi}{3}\right)$. Point the tail on your answer to (c).

(b) $\dfrac{x}{2} = \cos t$ so $\cos^2 t + \sin^2 t = 1$

$\dfrac{y}{-3} = \sin^2 t$ $\dfrac{x^2}{4} - \dfrac{y}{3} = 1$

so $\boxed{\dfrac{3x^2}{4} - 3 = y}$ (b)

(c) $\left(2\cos\frac{\pi}{3}, -3\sin^2\frac{\pi}{3}\right)$

$= \left(1, -3\left(\frac{\sqrt{3}}{2}\right)^2\right)$

$= \left(1, -\frac{9}{4}\right)$

(d) $\vec{v}(t) = -2\sin t\,\vec{i} - 6\sin t\cos t\,\vec{j} = -2\sin t\,\vec{i} - 3\sin 2t\,\vec{j}$

$\vec{v}\left(\frac{\pi}{3}\right) = -\frac{2\sqrt{3}}{2}\,\vec{i} - 6\frac{\sqrt{3}}{2}\cdot\frac{1}{2}\,\vec{j}$

$= -\sqrt{3}\,\vec{i} - \frac{6\sqrt{3}}{4}\,\vec{j}$

$\vec{a}(t) = -2\cos t\,\vec{i} - 6\cos 2t\,\vec{j}$

$\vec{a}\left(\frac{\pi}{3}\right) = -2\left(\frac{1}{2}\right)\vec{i} - 6\left(-\frac{1}{2}\right)\vec{j}$

$= -\vec{i} + 3\vec{j}$