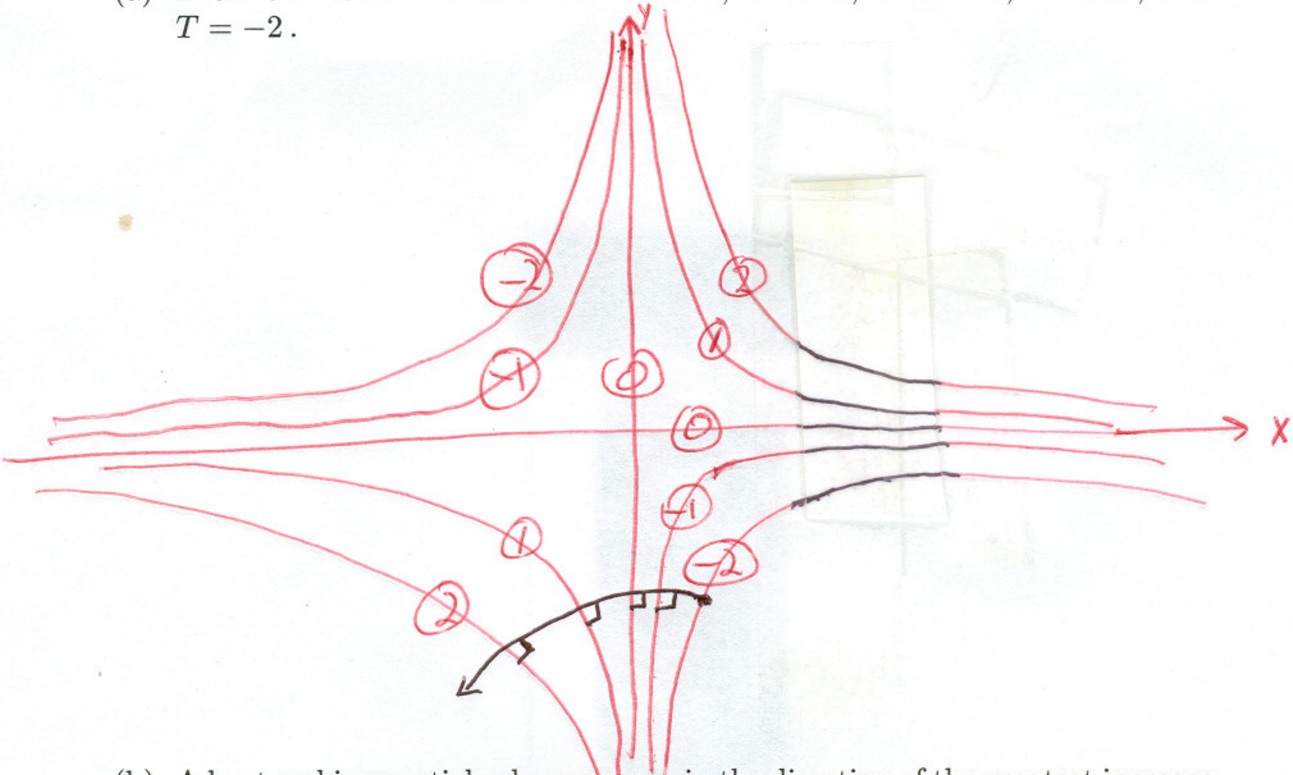
9. The temperature of a plate at the point (x, y) is $T(x, y) = xy$.
   (a) Draw and label the level sets $T = 0$, $T = 1$, $T = -1$, $T = 2$, and $T = -2$.



   (b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point $(1, -2)$. Draw the path of the particle.
   (c) Find the equation which gives the path of the particle of part (b).

Let $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ be the position vector of the object at time t. We know that

$\vec{r}'(t) = \kappa(t)\,\vec{\nabla}T|_t$ for some function $\kappa(t)$.

So $\vec{r}'(t) = \kappa(t)\left(y(t)\,\vec{i} + x(t)\,\vec{j}\right)$

$x'(t)\,\vec{i} + y'(t)\,\vec{j} = \kappa(t)\left(y(t)\,\vec{i} + x(t)\,\vec{j}\right)$

So $x'(t) = \kappa(t)\,y(t)$

$y'(t) = \kappa(t)\,x(t)$

That is $\dfrac{x'(t)}{y(t)} = \kappa(t) = \dfrac{y'(t)}{x(t)}$

In other words

$x(t)\,x'(t) = y(t)\,y'(t)$.

Integrate w.r.t t

$\int x(t)\,x'(t)\,dt = \int y(t)\,y'(t)\,dt$

$\dfrac{(x(t))^2}{2} + C = \dfrac{(y(t))^2}{2}$

at $t=0$ $x=1$ and $y=-2$

$\frac{1}{2} + C = \frac{4}{2}$

$C = 2 - \frac{1}{2} = \frac{3}{2}$

I will ignore t, because I don't know how fast the object travels.

$\dfrac{x^2}{2} + \dfrac{3}{2} = \dfrac{y^2}{2}$

$\boxed{x^2 + 3 = y^2}$