7. Find the directional derivative $D_{\vec{u}} f$ at $(1,2)$ for the function $f(x,y) = 3x^2y$ in the direction of the unit vector $\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$.

$D_{\vec{u}} f|_{(1,2)} = \vec{\nabla} f|_{(1,2)} \cdot \vec{u} = (6xy\vec{i} + 3x^2\vec{j})|_{(1,2)} \cdot (\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j})$

$= (12\vec{i} + 3\vec{j}) \cdot (\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j})$

$= \frac{36+12}{5} = \frac{48}{5}$

8. Find the length of the curve whose position vector is

$$\vec{r}(t) = t^2\,\vec{i} - 2t^3\,\vec{j} + 6t^3\,\vec{k},$$

for $0 \le t \le 1$.

length $= \int_0^1 \|\vec{r}\,'(t)\|\,dt = \int_0^1 \|2t\vec{i} - 6t^2\vec{j} + 18t^2\vec{k}\|\,dt = \int_0^1 2t\|\vec{i} - 3t\vec{j} + 9t\vec{k}\|\,dt$

$= \int_0^1 2t\sqrt{1+9t^2+81t^2}\,dt = \int_0^1 2t\sqrt{1+90t^2}\,dt = \frac{2}{3\cdot 90}(1+90t^2)^{\frac{3}{2}}\Big]_0^1$

$= \frac{2}{270}\left((91)^{\frac{3}{2}} - 1\right)$