5. Find the equation of the plane tangent to $z^2 = x^2 + y^2$ at the point $(3, 4, 5)$.

$\vec{\nabla}$'s are $\perp$ to level sets

We look at the level set $0 = x^2 + y^2 - z^2$

$\vec{\nabla}(RHS) = 2x\vec{i} + 2y\vec{j} - 2z\vec{k}$

$\vec{\nabla}(RHS)\big|_{(3,4,5)} = 6\vec{i} + 8\vec{j} - 10\vec{k}$

$6(x-3) + 8(y-4) - 10(z-5) = 0$

6. Consider the curve whose position vector is

$$\vec{r}(t) = 2t^2\,\vec{i} - t^3\,\vec{j} + \frac{4}{t}\,\vec{k}.$$

Find the equations of the line tangent to this curve at $t = 1$.

$\vec{r}(1) = 2\vec{i} - \vec{j} + 4\vec{k}$ So the point is $(2, -1, 4)$

$\vec{r}\,'(t) = 4t\vec{i} - 3t^2\vec{j} - \frac{4}{t^2}\vec{k}$

$\vec{r}\,'(1) = 4\vec{i} - 3\vec{j} - 4\vec{k}$

$$\frac{x-2}{4} = \frac{y+1}{-3} = \frac{z-4}{-4}$$