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10. Find the absolute extreme points of $f(x,y) = x^2 - 6x + y^2 - 8y + 7$ on $\{(x,y) \mid x^2 + y^2 \le 1\}$.

$f_x = 2x - 6$

$f_y = 2y - 8$

$f_x = f_y = 0$

at $x=3$ $y=4$

$(3,4)$ is not in the domain.

The max and min occur on the boundary $x = \cos\theta$, $y = \sin\theta$.

$f(\theta) = \cos^2\theta - 6\cos\theta + \sin^2\theta - 8\sin\theta + 7$

$f(\theta) = -6\cos\theta - 8\sin\theta + 8$ ⟵ this is f on the boundary.

$\cos^2\theta + \sin^2\theta = 1$

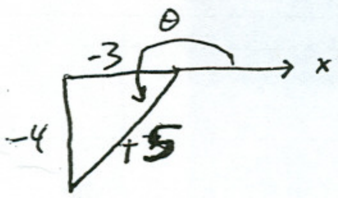
$f'(\theta) = +6\sin\theta - 8\cos\theta$

$f'(\theta) = 0$ when $6\sin\theta - 8\cos\theta = 0$

$$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$$

so $\tan\theta = \frac{4}{3}$

so either



Thus $\sin\theta = \frac{4}{5}$ $\cos\theta = \frac{3}{5}$ or $\sin\theta = \frac{-4}{5}$ $\cos\theta = -\frac{3}{5}$

This is the point

$(\frac{3}{5}, \frac{4}{5})$, $f(\frac{3}{5}, \frac{4}{5}) = \frac{9}{25} - \frac{18}{5} + \frac{16}{25} - \frac{32}{5} + 7 = \frac{-50}{5} + 8 = -2$ ⟸ min

$f(-\frac{3}{5}, -\frac{4}{5}) = \frac{9}{25} + \frac{18}{5} + \frac{16}{25} + \frac{32}{5} + 7 = \frac{50}{5} + 8 = 18$ ⟸ max

Max $(-\frac{3}{5}, -\frac{4}{5}, 18)$

Min $(\frac{3}{5}, \frac{4}{5}, -2)$