

6. Identify all local extreme points and all saddle points of $f(x, y) = x^3 + y^3 - 6xy$.

$$f_x = 3x^2 - 6y$$

$$f_y = 3y^2 - 6x$$

$$f_x = f_y = 0 \text{ when}$$

$$0 = x^2 - 2y$$

$$0 = y^2 - 2x$$

$$\text{So } x = \frac{y^2}{2} \text{ (from bottom eq)}$$

$$0 = \left(\frac{y^2}{2}\right)^2 - 2y$$

$$0 = y^4 - 8y$$

$$0 = y(y^3 - 8)$$

$$\text{so } y = 0 \text{ or } y = 2$$

$$\text{but } x = \frac{y^2}{2}$$

so our critical points

are $(0, 0)$ and $(2, 2)$

$$f_{xx} = 6x$$

$$f_{yy} = 6y$$

$$f_{xy} = -6$$

at $(0, 0)$

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

$$= 0 \cdot 0 - 36 < 0$$

Thus $(0, 0, 0)$ is a saddle point

at $(2, 2)$

$$D = 12 \cdot 12 - 36 > 0$$

$$f_{xx}|_{(2,2)} = 12 > 0$$

$(2, 2, f(2, 2))$ is a local min

7. Graph $z = 9 - y^2 - x^2$ in 3-space.

